FRIEDRICH-ALEXANDER UNIVERSITÄT ERLANGEN-NÜRNBERG
FACULTY OF BUSINESS, ECONOMICS, AND LAW

partialCI: An R Package for the Analysis of Partially Cointegrated Time Series

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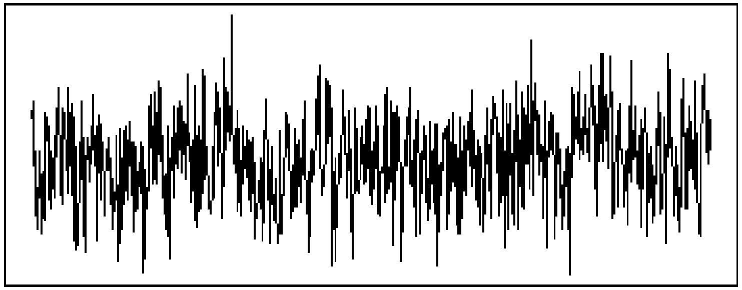
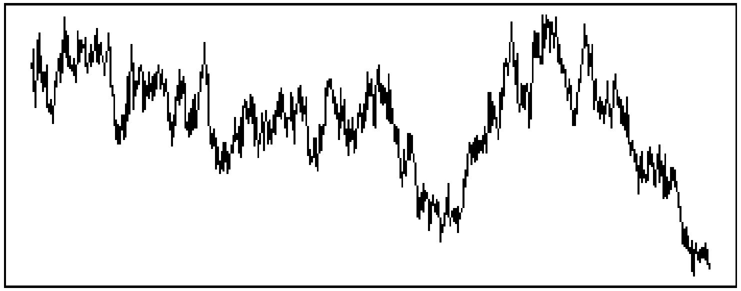
Objectives and key issues to be covered today

- **What is partial cointegration?**
- **How does it work?**
- **How to use partialCI?**
- **Where is it helpful?**

What is partial cointegration (PCI)? – A weakening of cointegration allowing for transient and permanent components in the residual series [1]

	Classic cointegration[2]	Partial cointegration
Residual series	Consists of a **stationary** mean-reverting component	Consists of a sum of a **permanent** and a **stationary** mean-reverting component (partially autoregressive (PAR) [3] process)
Shocks	Are required to be **transient**	Are allowed to be **transient** and **permanent**
Visual-ization		

[1] Clegg and Krauss (2016) [2] Engle and Granger (1987) [3] Poterba and Summers (1988), Clegg (2015)

How does it work? – A primer on the methodology

The partial cointegration framework[4]

- Y_t and $X_{j,t}$ are partially cointegrated, if a parameter vector $\iota = \{\beta, \rho, \sigma_M, \sigma_R\}$ exists so that:

$$Y_t = \beta_1 X_{1,t} + \beta_2 X_{2,t} + \ldots + \beta_k X_{k,t} + W_t$$
$$W_t = M_t + R_t$$
$$M_t = \rho M_{t-1} + \varepsilon_{M,t}; \varepsilon_{M,t} \sim \mathcal{N}\left(0, \sigma_M^2\right)$$
$$R_t = R_{t-1} + \varepsilon_{R,t}; \varepsilon_{R,t} \sim \mathcal{N}\left(0, \sigma_R^2\right)$$
$$\beta_j \in \mathbb{R}; \rho \in (-1,1); \sigma_M^2, \sigma_R^2 \in \mathbb{R}_0^+$$

- Proportion of variance attributable to mean reversion (PVMR):

$$R_{MR}^2 = \frac{VAR\left[(1-B) M_t\right]}{VAR\left[(1-B) W_t\right]} = \frac{2\sigma_M^2}{2\sigma_M^2 + (1+\rho)\sigma_R^2}$$

Notation / Assumptions

- Notation
  - Target time series: Y_t
  - Factor time series j: $X_{j,t}, j = 1, \ldots, k$
  - Coefficient of factor time series j: β_j
  - PAR process: W_t
  - Mean-reverting component: M_t
  - Permanent component: R_t
  - Error terms M_t , R_t: $\varepsilon_{M,t}$, $\varepsilon_{R,t}$
  - Vector of factor coefficients: β
  - Coefficient of mean-reversion: ρ
  - PVMR: R_{MR}^2

- Assumptions: $\varepsilon_{M,t}$ and $\varepsilon_{R,t}$ are mutually independent normally distributed white noise processes with mean zero and variances σ_M^2 and σ_R^2

[4] Clegg and Krauss (2016)

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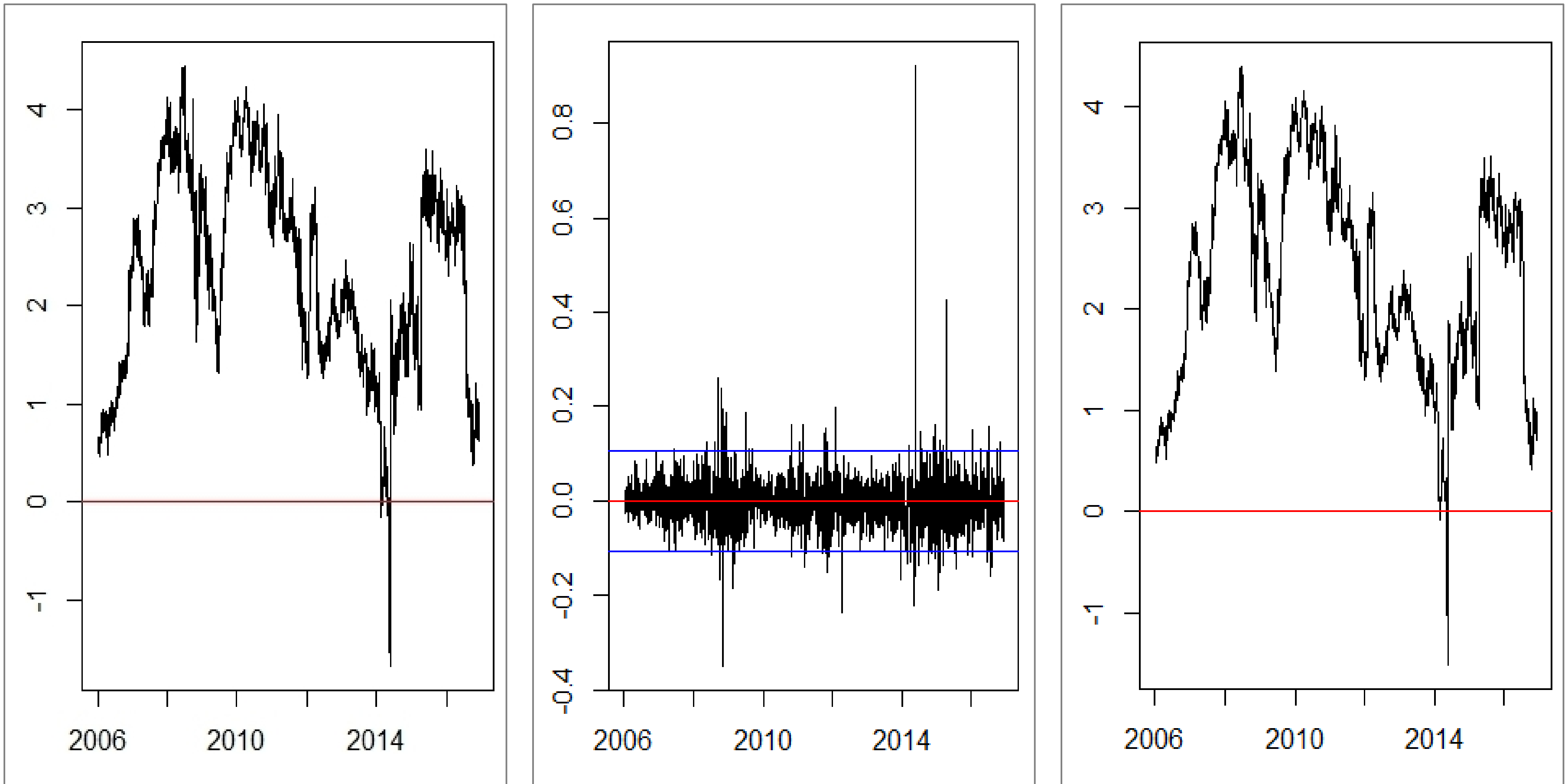
How to use partialCI? – An overview of the key functions

Function	Description	Code
fit.pci	Fits a partial cointegration model to a given collection of time series	`fit.pci(Y, X, pci_opt_method = c("jp", "twostep"),par_model = c("par", "ar1", "rw"), lambda = 0, robust = FALSE, nu = 5, include_alpha=FALSE)`
test.pci	Tests the goodness of fit of a PCI model	`test.pci(Y, X, alpha = 0.05, null_hyp = c("rw", "ar1"),robust = FALSE, pci_opt_method = c("jp", "twostep"))`
statehistory.pci	Estimates and extracts the sequence of hidden states	`statehistory.pci(A, data = A$data, basis = A$basis)`
hedge.pci	Finds k factors from a predefined set of factors which yield the best fit to the target time series	`hedge.pci(Y, X,use.multicore = TRUE, minimum.stepsize = 0, exclude.cols = c(), search_type = c("lasso", "full", "limited"),pci_opt_method=c("jp","twostep"))`

Where is it helpful (1/2)? – An example in pairs trading: RDS-A and RDS-B (2006-01-01 – 2016-01-12, daily prices)[5]

Spread: Classic cointegration	Spread: Transient component	Spread: Random walk component



[5] Data and R-code: https://github.com/jonasrende/Rfinance2017

Where is it helpful (2/2)? – The test indicates that RDS-A and RDS-B are indeed partially cointegrated

Fit a PCI model

```
PCI_RDSA_RDSB<-fit.pci(RDSA, RDSB,
pci_opt_method = c("jp"),
par_model =c("par"), lambda = 0,
robust = FALSE, nu = 5,
include_alpha = FALSE))

Fitted values for PCI model
Y[t] = X[t] %*% beta + M[t] + R[t]
M[t] = rho * M[t-1] + eps_M [t],
          eps_M[t] ~ N(0, sigma_M^2)
R[t] = R[t-1] + eps_R [t],
          eps_R[t] ~ N(0, sigma_R^2)

            Estimate Std. Err
beta_Close 0.9264   0.0038
rho        0.3959   0.0965
sigma_M    0.1063   0.0082
sigma_R    0.1174   0.0074

-LL = -1165.16, R^2[MR] = 0.540
```

Test for PCI

```
test.pci(RDSA, RDSB, alpha = 0.05,
null_hyp = c("rw", "ar1"), robust =
FALSE, pci_opt_method = c("jp"))

Likelihood ratio test of [Random Walk
or CI(1)] vs Almost PCI(1)
(joint penalty method)

data: RDSA

Hypothesis      Statistic p-value
Random Walk        -55.09   0.010
AR(1)              -52.88   0.010
Combined                    0.010
```

Interested? – Further references

Paper / Package	Title	URL	QR code
Full paper	partialCI: An R package for the analysis of partially cointegrated time series	http://hdl.handle.net/10419/150014	
Slides	partialCI: An R package for the analysis of partially cointegrated time series	https://www.statistik.rw.fau.de/files/2017/05/v01-2017.pdf	
R package (CRAN)	partialCI: Partial Cointegration	https://cran.r-project.org/web/packages/partialCI/index.html	
Initial show case for partial cointegration	Pairs trading with partial cointegration	http://hdl.handle.net/10419/140632	