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A Note on Conditional Arbitrage-Free Maximum Entropy Densities For Simulative Option Pricing

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SUMMARY

In this note we present a simple method to include the no-arbitrage condition into the derivation of conditional densities using the principle of maximum entropy. For the case of identically and independently distributed returns, we easily derive that the whole process estimated that way is arbitrage free. Such a process may be directly used for simulative derivation of option prices.

Keywords and phrases: Maximum Entropy density; No Arbitrage Condition

1 Introduction

In this note we present a simple method to include the no-arbitrage condition into the derivation of conditional densities using the principle of maximum entropy. For the case of identically and independently distributed returns, we easily derive that the whole process estimated that way is arbitrage free. Such processes may be directly used for simulative derivation of option prices.

In the first section we present the processes under consideration. In section 2 we show that the whole process is arbitrage-free. Section 3 presents some selected models and section 4 its results for different call options on the DAX30 compared to Black/Scholes and observed option prices.

2 Maximum Entropy Models for Assets Returns

Let P_t be the process of asset prices P at time t with

$$P_{t+1} = P_t \cdot e^{R_t},$$

such that R_t is its log-return at time t, with $t = 1, 2, ..., \tau$. In this note we will only consider processes where the conditional distribution of R_t given the information available at time $\mathfrak{F}_t$ is given as the distribution maximizing some generalized entropy measure

$$H(f) = \int_{D(R_t)} \phi(f(r_t)) dr_t,$$

where $D(R_t)$ is the support of the returns and ϕ some convex function, subject to the constraints that

$$f(r_t) > 0 \quad \forall r_t \in D(R_t), \quad E(1) = \int_{D(R_t)} f(r_t) dr_t = 1,$$

to have a proper density function, additionally we assume that

$$E(R_t^2) = \int_{D(R_t)} r_t^2 f(r_t) dr_t = \sigma,$$

to set variance to a fixed value. Contrary to existing approaches we also include knowledge from the no-arbitrage condition as

$$E(\frac{P_{t+1}}{P_t}) = E(\exp(R_t)) = \int_{D(R_t)} \exp(r_t) f(r_t) dr_t = \exp(r_{f,t}),$$

where $r_{f,t}$ denotes the risk free overnight rate a time t.

For given moment constraints the resulting density function may only be derived numerically by minimizing a corresponding dual problem. For the numerical implementation of such a dual problem we refer to Rockinger/Jondeau (2002).

3 Arbitrage Free Processes

Assuming that σ, m_3, m_4 and the risk free overnight interest rate $r_{r,t}$ as a deterministic function of the time, the R_t are independently distributed. But then it holds for the whole process from $t = 1..\tau$ that

$$E\left(\exp(\sum_{i=1}^{\tau} R_i)\right) = \prod_{i=1}^{\tau} E\left(\exp(R_i)\right) = \prod_{i=1}^{\tau} exp(r_{f,t}) = \exp(r_f)$$

where r_f denotes the risk free interest rate from time 1 to time τ. Because the no-arbitrage condition for the process from time 1 to τ is

$$E\left(\frac{P_{1+\tau}}{P_1}\right) = E\left(exp(\sum_{i=1}^{\tau} R_i)\right) = \exp(r_f),$$

we find that the whole process is arbitrage free.

4 Simulative Option Prices for the DAX30

We use the last 50 daily observations to estimate our model and also to determine the variance for the Black/Scholes formula. We try to explain option prices for the DAX30 on June 12 between 19:59 and 20:00. We consider options with different strike prices that all expire on August 19. Table(1) shows some results:

We conclude that our model gives similar results as the Black/Scholes model, but is not able to explain observed option prices better.

	3000	3500	4000	4500	5000	5500	6000
Observed	20.77	15.83	10.98	6.45	2.70	0.63	0.08
Black/Scholes	20.77	15.79	10.83	6.19	2.68	0.84	0.19
Our Model	20.70	15.70	10.73	6.09	2.62	0.80	0.18

Table 1: Estimated option prices in Euro from simulation based on our model compared to observed option prices and to results from the Black/Scholes formula.

5 Summary

We propose a simple framework to include the no-arbitrage condition in the estimation of the process of financial markets returns. In a very basic setting, we use this framework to simulate option prices and find results similar to those from the Black/Scholes formula. The model might be improved by using overnight interest rates as instrument for the daily risk-free interest rate.

References

[1] Rockinger, M. and E. Jondeau (2002): "Entropy Densities with an Application to Autoregressive Conditional Skewness and Kurtosis". *Journal of Econometrics*, **106**:119-142.