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Abstract

We develop a copula-based pairs trading framework and apply it to the S&P 100 index constituents from 1990 to 2014. We propose an integrated approach, using copulas for pairs selection and trading. Essentially, we fit t-copulas to all possible combinations of pairs in a 12 month formation period. Next, we run a 48 month in-sample pseudo-trading to assess the profitability of mispricing signals derived from the conditional marginal distribution functions of the t-copula. Finally, the most suitable pairs based on the pseudo-trading are determined, relying on profitability criteria and dependence measures. The top pairs are transferred to a 12 month trading period, and traded with individualized exit thresholds. In particular, we differentiate between pairs exhibiting mean-reversion and momentum effects and apply idiosyncratic take-profit and stop-loss rules. For the top 5 mean-reversion pairs, we find out-of-sample returns of 7.98 percent per year; the top 5 momentum pairs yield 7.22 percent per year. Return standard deviations are low, leading to annualized Sharpe ratios of 1.52 (top 5 mean-reversion) and 1.33 (top 5 momentum), respectively. Since we implement this strategy on a highly liquid stock universe, our findings pose a severe challenge to the semi-strong form of market efficiency and demonstrate a sophisticated yet profitable alternative to classical pairs trading.

Keywords: Statistical arbitrage, pairs trading, quantitative strategies, copula.

JEL Classification: G11, G12, G14.

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1. Introduction

At first sight, pairs trading is supposedly very simple: Find two stocks that move together historically. In case of price divergence, short the winning stock and buy the losing stock. If history repeats itself, the prices converge and a profit can be made. The seminal paper of Gatev et al. (2006) on that subject exhibits raw returns of 11 percent p.a. on the U.S. stock universe from 1962-2002, albeit with declining profitability in the more recent part of the sample. Do and Faff (2010) confirm these findings. Ever since, academic interest in pairs trading has surged considerably. As of today, there exists a plethora of different approaches to statistical arbitrage pairs trading. A comprehensive literature overview can be found in Krauss (2015). In this paper, we focus on the copula approach to pairs trading. Generally speaking, mispricings and ultimately, trading signals, are determined using copulas - a powerful tool for dependence modeling. Two substreams have developed in the literature, return-based versus level-based copula methods. These approaches shall be briefly described here below:

Return-based copula method: Following Krauss (2015), representatives of this category are Ferreira (2008), Liew and Wu (2013), and Stander et al. (2013). In a formation period, pairs are created with commonly applied comovement metrics, i.e., correlation or cointegration criteria. Then, we consider the log returns $R_1 = (R_{1,t})_{t\in T}$ and $R_2 = (R_{2,t})_{t\in T}$ for the two legs 1 and 2 of each pair and estimate their marginal distributions F_{R_1} and F_{R_2}. Ferreira (2008) and Liew and Wu (2013) fit parametric distribution functions, whereas Stander et al. (2013) discuss parametric and non-parametric approaches to obtain the marginal distributions instead. Probability integral transform by plugging the returns into their own marginals creates the uniform variables $U_1 = F_{R_1}(R_1)$ and $U_2 = F_{R_2}(R_2)$. At this stage, an appropriate copula function can be identified. Ferreira (2008) uses one Archimedean copula, whose parameters are estimated via canonical maximum likelihood. Conversely, Stander et al. (2013) employ 22 different Archimedean copulas and determine the best-fit with the Kolmogorov-Smirnov goodness-of-fit test. Liew and Wu (2013) start with five commonly used copulas in financial applications and determine the best-fitting one by evaluating Akaike, Bayesian and Hannan-Quinn information criteria. The trading strategy is similar across all three authors and is described following Stander et al. (2013) and Liew and Wu (2013). Both authors first calculate the conditional marginal distribution functions as first partial derivatives of the copula function $C(u_1, u_2)$ of the best-fitting parametric copula:

$$h_1(u_1|u_2) = P(U_1 \leq u_1|U_2 = u_2) = \frac{\partial C(u_1, u_2)}{\partial u_2}, \quad (1)$$
$$h_2(u_2|u_1) = P(U_2 \leq u_2|U_1 = u_1) = \frac{\partial C(u_1, u_2)}{\partial u_1}.$$

In case the conditional probability is higher (lower) than 0.5, a stock can be considered overvalued (undervalued) relative to its peer. The authors trade when the conditional probabilities are well in the tail regions of the conditional distribution functions, i.e., below the 5 percent and above the 95 percent confidence level. To be specific, stock 1 is bought and stock 2 is sold short when the pair of transformed returns falls outside both confidence bands derived by $P(U_1 \leq u_1|U_2 = u_2) = 0.05$ and $P(U_2 \leq u_2|U_1 = u_1) = 0.95$. Informally speaking, an entry signal occurs when the transformed returns fall in the extreme regions in the northwest quadrant of a scatter plot of U_1 and U_2. Conversely, stock 2 is bought and stock 1 sold short, in case inverse conditions apply (extreme regions in the southeast quadrant). Stander et al. (2013) suggest to exit a trade as soon as it is profitable or after one trading week. Liew and Wu (2013) reverse the positions once the conditional probabilities cross the boundary of 0.5 a second time.

This approach is appealing, as copulas allow for nonlinear dependence modeling. An improved modeling of stylized facts of financial return data, such as negative skewness and excess kurtosis, see Cont (2001), may also lead to the identification of more reliable trading opportunities. However, three issues remain. First, pair selection is not copula-based, but the top 20 pairs with maximum Spearman's ρ are chosen, introducing a selection bias. Second, the time structure of the data is completely lost, meaning that copula-based entry and exit signals are only anchored on the last return without assessing how each pair trades subsequent to such copula-based entry signals, i.e., convergence vs. further divergence. Third, the approach has only been tested on very few securities and not on a large and liquid stock universe.

Level-based copula method: Xie and Wu (2013), Xie et al. (2014), and Rad et al. (2015) develop the level-based copula method. Essentially, the authors define mispricing as conditional probabilities in (1) minus 0.5. These mispricings are accumulated over several periods to a mispricing index, thus reflecting how far the securities are out of equilibrium. Their approach is appealing, as it reflects multi-period mispricings and thus better retains the time structure, compared to the return-based copula method. However, there are a few downsides associated with it: First, pair selection is not copula-based, but the top 20 pairs with minimum sum of squared distances are chosen, introducing a selection bias. Clearly, the copula method in Rad et al. (2015) is merely an alternative way of determining entry and exit signals but has no influence on pairs formation. Second, the authors rely on the theoretical framework of Xie et al. (2014), who derive the conditions under which the cumulative mispricing indices are mean-reverting. However, Rad et al. (2015) do not test if the cumulative mispricing indices really are mean-reverting - a quintessential prerequisite for profitable pairs trading. Third, no differentiation is made between pairs reaching critical levels of mispricing indices over time by aggregating many small mispricings versus pairs reaching the critical levels in a few large steps.

This paper proposes an integrated return-based copula approach and shows an empirical application to the constituents of the S&P 100 from 1990-2014. We run a copula-based pairs selection, taking into account a 60 month formation period, consisting of overlapping 12 month estimation and subsequent 1 month pseudo-trading periods. The most attractive pairs based on profitability and a minimum dependence criterion are transferred to a 12 month out-of-sample trading period. We apply individualized trading rules to each pair, equally derived during the pseudo-trading period. In particular, we differentiate between "mean-reversion pairs" and "momentum pairs". The former exhibit a tendency to drift further apart after an entry-signal in the pseudo-trading period, the latter exhibit a tendency for mean-reversion. The pairs are traded accordingly during the actual 12 month out-of-sample trading period. Likewise, suitable individualized stop-loss and take-profit levels for the out-of-sample trading period are developed based on the aggregated cumulative return of each pair, conditional to its copula-based entry signals.

For the top 5 mean-reversion pairs, we find that our copula-based pairs trading strategy produces statistically significant raw returns of 7.98 percent per year at very low standard deviation, resulting in an annualized Sharpe ratio of 1.52. The top 5 momentum pairs yield similar results with annualized raw returns of 7.22 percent per year and a Sharpe ratio of 1.33. Considering the top 10 pairs, we only observe a slight deterioration. We show that returns only marginally load on common systematic sources of risk. The incorporation of market frictions in form of a one-day-waiting rule and trading costs of 0.50 percent per round-trip trade per pair negatively affects the results, but annualized returns remain above 5 percent per annum. We find that the copula-based pairs trading strategy exhibits low drawdown risk compared to a naive buy-and-hold strategy in the S&P 100. Particularly, the maximum drawdown for the top 5 mean-reversion and top 5 momentum pairs is below 10 percent, compared to more than 50 percent for the S&P 100. Our findings are similar for other measures of tail risk, thus potentially allowing for the application of higher leverage than 2:1 - the standard in pairs trading. Considering that we implement this strategy on the highly liquid stocks of the S&P 100, our findings contribute to the literature in several respects. First, we make a methodological contribution by introducing an integrated copula-based pairs selection and trading strategy with per-pair individualized trading rules. Particularly, we consider suitable stop-loss and take-profit levels, we differentiate between mean-reversion and momentum pairs, and we take into account the time dependency of the return time series via conditional cumulative return time series after the entry signals in the pseudo-trading period. Second, building on these foundations, we show that this sophisticated pairs trading strategy is profitable even in a highly efficient market and in recent years. Conversely, classical pairs trading as in Gatev et al. (2006) exhibits declining profitability, see Do and Faff (2010). Third, we may conclude that our findings pose a severe challenge to the semi-strong form of market efficiency, underlining the potential of refined relative-value arbitrage strategies.

The rest of this paper is organized as follows: Section 2 covers the data sample used in this study. Section 3 provides an in-detail description of the methodology. Section 4 presents the results and discusses key findings in light of the existing literature. Finally, section 5 concludes and summarizes directions for further research.

2. Data and software

2.1 Data

For our empirical application, we opt for the S&P 100. Our choice is motivated by two determinants, market efficiency and computational feasibility. The S&P 100 consists of the leading 100 blue-chips in the U.S. stock market, accounting for approximately 50 percent of total U.S. market capitalization (S&P Dow Jones Indices, 2015). This highly liquid subset of the stock market serves as a true acid test for any trading strategy, as investor scrutiny and analyst coverage is especially high for these large capitalization stocks. Conversely, handling merely 100 constituents per month renders even sophisticated strategies computationally feasible, making this index the universe of choice for our strategy. We proceed as follows for eliminating the survivor bias. First, we obtain all month end constituent lists for the S&P 100 from Thomson Reuters Datastream from December 1989 to November 2014. We consolidate these lists into a binary matrix, showing a "1", if the stock is a constituent of the index in the subsequent month and a "0" otherwise. Second, for all stocks having ever been a constituent of the index, we download the daily total return indices (RI) from January 1990 until December 2014. The RI reflects cum-dividend prices and accounts for all further corporate actions and other effects, such as stock splits, making it the most suitable metric for return calculations. Previously reported concerns about Datastream quality in Ince and Porter (2006) are mainly concentrated on the smaller size deciles. Also, Datastream seems to have reacted to the objections in the meantime, see Leippold and Lohre (2012). Hence, besides eliminating holidays, we apply no further measures for data sanitization.

For some preliminary analyses on the suitability of different bivariate copula models for financial return data, we prepare a further out-of-sample data set based on the DAX 30 constituents as of December 2014. We obtain return index data from January 2005 until December 2014.

2.2 Software

The entire methodology in this paper is implemented in the statistical software `R`, relying on the packages depicted in table 1.

Application	**R package**	**Authors of the R package**
Copula modeling	`copBasic`	Asquith (2015)
Copula modeling	`copula`	Hofert et al. (2015)
Copula modeling	`fCopulae`	Würtz and Setz (2014)
Data handling	`dpylr`	Wickham and Francois (2015)
Data handling	`xts`	Ryan and Ulrich (2014)
Financial modeling	`PerformanceAnalytics`	Peterson and Carl (2014)
Financial modeling	`QRM`	Pfaff and McNeil (2014)
Financial modeling	`quantmod`	Ryan (2015)
Financial modeling	`tseries`	Trapletti and Hornik (2015)
Financial modeling	`TTR`	Ulrich (2013)
General modeling	`sandwich`	Zeileis (2006)
General modeling	`stats`	R Core Team (2015)

Table 1: R packages used in this paper.

3. Methodology

3.1 Preliminaries

3.1.1 Copula concept: Following Nelsen (2006), any function $C : [0,1]^n \to [0,1]$ is called an n-dimensional copula (n-copula) if the following three properties are satisfied:

1. $\forall u = (u_1, \ldots, u_n) \in [0,1]^n\colon \min\{u_1, \ldots, u_n\} = 0 \implies C(u) = 0$,
2. $C(1, \ldots, 1, u_i, 1, \ldots, 1) = u_i \quad \forall u_i \in [0,1] \quad (i \in \{1, \ldots, n\})$,
3. $V_C([a,b]) \geq 0$, where $V_C([a,b])$ denotes the C-volume of the hyperrectangle $[a,b] = \prod_{i=1}^{n} [a_i, b_i]$, $a_i \leq b_i \quad \forall i \in \{1, \ldots, n\}$.

A copula establishes a functional relationship between a multivariate distribution function and its marginals, as expressed in Sklar's theorem (Sklar, 1959): Let $F_{X_1,\ldots,X_n}$ be an n-dimensional distribution function with marginal distributions F_{X_i} $(i = 1, \ldots, n)$. Then, there exists an n-copula C which satisfies the following equation for all $(x_1, \ldots, x_n) \in \mathbb{R}^n$:

$$F_{X_1,\ldots,X_n}(x_1, \ldots, x_n) = C(F_{X_1}(x_1), \ldots, F_{X_n}(x_n)). \tag{2}$$

If the margins are continuous, then C is unique. Conversely, let C be an n-copula and $F_{X_1}, \ldots, F_{X_n}$ univariate distribution functions. Then, the function $F_{X_1,\ldots,X_n}$ defined by equation (2) is an n-dimensional distribution function with marginal distribution functions $F_{X_1}, \ldots, F_{X_n}$.

3.1.2 Goodness-of-fit of copulas In this section, we decide on one copula we use for modeling the dependence structure of each pair in our empirical application, following the methodology outlined in Genest and Rivest (1993), Genest et al. (1995), and Genest et al. (2009). Specifically, we compare five Archimedean copulas (Ali-Mikhail-Haq, Clayton, Frank, Gumbel, Joe), two elliptical copulas (Gaussian, Student's t) and four extremal value copulas (Galambos, Hüsler-Reiss, Tawn and t-EV) regarding their goodness-of-fit to financial return data. In order to avoid data-snooping bias, we run this analysis on a different set of liquid stocks, namely the DAX 30 constituents as of December 2014. We form $30(30-1)/2 = 435$ bivariate information sets in total, using the return time series from January 2005 until December 2014 as input for each stock, thus providing us with more than 2000 observations for reliable estimates. Each copula is fitted to each of these 435 pairs, and we construct two rankings, one relying on the Cramér-von Mises test, one relying on information criteria:

Cramér-von Mises test: Following Genest et al. (2009), let $\mathcal{C}_0 = \{C_\theta : \theta \in \Theta\}$ be a class of copulas, where the parameter set Θ is an open subset of $\mathbb{R}^p$ ($p \geq 1$). An unknown copula C belongs to $\mathcal{C}_0$ if $C \in \mathcal{C}_0$. The goodness-of-fit test considers the null hypothesis $H_0 : C \in \mathcal{C}_0$. Modeling the marginals by parametric distributions is no longer necessary. Following Deheuvels (1978), the empirical copula is defined by

$$C_n(u) = \frac{1}{n} \sum_{i=1}^{n} \mathbb{1}\{U_{i1} \leq u_1, \ldots, U_{id} \leq u_d\}, \quad u = (u_1, \ldots, u_d) \in [0,1]^d.$$

Let C_{θ_n} be the estimation of C assuming that C belongs to $\mathcal{C}_0$. Similar to the goodness-of-fit tests for univariate distributions, the goodness-of-fit tests for copulas are based on the distance between C_n and C_{θ_n}:

$$\mathbb{C}_n = \sqrt{n}(C_n - C_{\theta_n}),$$

which is minimized for "good" estimators. Genest et al. (2009) suggest the following statistics:

- Cramér-von Mises statistic: $S_n = \int\limits_{[0,1]^d} \mathbb{C}_n(u)^2 dC_n(u)$,
- Kolmogorov-Smirnov statistic: $T_n = \sup\limits_{u \in [0,1]^d} |\mathbb{C}_n(u)|$.

Large values of these statistics result in the rejection of the null hypothesis. Genest and Rémillard (2008) show the consistency of both tests adapted from S_n and T_n, whereby $C \notin \mathcal{C}_0$ leads to a rejection of H_0 with a probability of 1 for $n \to \infty$. Genest et al. (2009) report the Cramér-von Mises functional of a process to be more powerful than the corresponding Kolmogorov-Smirnov statistic for the same process. Hence, we only consider the former statistic in our study. Genest et al. (2009) find that the asymptotic distribution of S_n cannot be determined, as it depends on the family of copulas and θ_n under the composite null hypothesis. Instead, they suggest a specific parametric bootstrap procedure, which we use for our tests. Table 2 depicts the results for the Cramér-von Mises test. The column "Average" denotes the average rank a copula achieves, ranging from 1-11, the column "Winner" denotes the empirical probability for each copula to achieve first rank. We see that the t-copula achieves an average rank of 1.23 across all 435 pairs, and ranks first in 71.26% of all cases, rendering it clearly superior to all other choices.

	Average	Winner
Archimedean copula		
Ali-Mikhail-Haq	6.55	4.60%
Clayton	6.40	0.69%
Frank	4.26	4.37%
Gumbel	7.50	0.92%
Joe	10.51	0.00%
Elliptical copula		
Gaussian	2.63	9.20%
Student's t	**1.23**	**71.26%**
Extreme value copula		
Galambos	6.66	2.35%
Hüsler-Reiss	8.85	0.00%
Tawn	4.06	5.52%
t-EV	7.35	1.15%

Table 2: Results for Cramér-von Mises goodness-of-fit test. ”Average” denotes the average rank a copula achieves, ranging from 1-11. ”Winner” denotes the empirical probability for each copula to achieve first rank.

Information criteria: We repeat the latter analysis based on information criteria, relying on the semi-parametric pseudo-maximum likelihood approach. First, we describe the marginals by their empirical cumulative distribution functions $\hat{F}_{X_i}$ ($i \in \{1, \dots, d\}$). Then, we plug them into the copula density yielding the log likelihood function with parameter set θ:

$$l(\theta) = -\sum_{i=1}^{n} \log[c_\theta(\hat{F}_{X_1}(x_{i,1}), \dots, \hat{F}_{X_d}(x_{i,d})].$$

Using $l(\theta)$, we introduce criteria which include penalty terms for the number of the copula parameters k. Akaike (1973) defines the Akaike Information Criterion (AIC) as follows:

$$AIC = -2l(\theta) + 2k.$$

The Bayesian Information Criterion (BIC), developed by Schwarz (1978), leads to a similar expression, resulting in higher penalties if $n > e^2$:

$$BIC = -2l(\theta) + \log(n)k.$$

The goodness-of-fit for the eleven copulas based on AIC and BIC are presented in table 3. Yet again, the t-copula is the dominant choice, with an average rank of 1.21 for AIC and 1.32 for BIC. It turns out to be the most suitable choice in more than 65% of all cases.

	AIC		BIC	
	Average	Winner	Average	Winner
Archimedean copula				
Ali-Mikhail-Haq	6.53	4.60%	6.51	4.60%
Clayton	3.83	2.53%	3.72	2.99%
Frank	6.21	3.22%	6.17	3.68%
Gumbel	4.32	6.21%	4.28	6.21%
Joe	10.31	0.00%	10.26	0.00%
Elliptical copula				
Gaussian	3.54	6.90%	3.52	7.82%
Student's t	**1.21**	**66.21%**	**1.32**	**65.06%**
Extreme value copula				
Galambos	7.26	1.38%	7.22	1.61%
Hüsler-Reiss	8.83	0.00%	8.79	0.00%
Tawn	4.86	4.83%	4.95	3.68%
t-EV	5.27	4.14%	5.25	4.37%

Table 3: Results for goodness-of-fit based on AIC and BIC. "Average" denotes the average rank a copula achieves, ranging from 1-11. "Winner" denotes the empirical probability for each copula to achieve first rank.

The salient point of these analyses is that the t-copula seems to be a sound choice for financial return data. This view is also supported by other authors, see Mashal et al. (2003), Breymann et al. (2003), and Kole et al. (2005). Since computational run times for repeated goodness-of-fit testing are high, we make a judgment call and fix the t-copula as model of choice for all further pairs for our S&P 100 application.

3.2 Formation period

The formation period T_f has a total length of 60 months. It is split in a 12 month estimation period T_{fe} and a subsequent 1 month pseudo-trading period T_{ft}. Following Jegadeesh and Titman (1993), we run the pseudo-trading in portfolios with a 1 month overlap, resulting in 48 sets of 12 month estimation and subsequent 1 month pseudo-trading periods in each 60 month formation period. Results are aggregated across overlapping portfolios.

3.2.1 Estimation period: In the 12 month estimation period, we form all possible combinations of $n(n-1)/2$ pairs. We follow Patton (2012) and use a semiparametric approach to fit bivariate t-copulas to these bivariate information sets. In the nonparametric step, for the two legs 1 and 2 of each pair, we compute the empirical marginal distribution functions F_{R_1} and F_{R_2} for the log return time series R_1 and R_2. Relying on the empirical marginals avoids model risk introduced by mis-specified parametric marginals. We use the empirical distributions to calculate scaled ranks for all returns, effectively creating uniformly distributed variables $U_1 = F_{R_1}(R_1)$ and $U_2 = F_{R_2}(R_2)$. In the parametric step, we fit a t-copula to the transformed returns U_1 and U_2, where the bivariate t-copula $C_{\rho,\nu}$ is defined as

$$
\begin{aligned}
C_{\rho,\nu}(u_1,u_2) &= t_{\rho,\nu}(t_\nu^{-1}(u_1), t_\nu^{-1}(u_2)) \\
&= \int_{-\infty}^{t_\nu^{-1}(u_1)} \int_{-\infty}^{t_\nu^{-1}(u_2)} \frac{\Gamma(\frac{\nu+2}{2})}{\Gamma(\frac{\nu}{2})\sqrt{(\pi\nu)^2(1-\rho)^2}} \left(1 + \frac{s^2+t^2-2\rho st}{\nu(1-\rho^2)}\right)^{-\frac{\nu+2}{2}} dtds,
\end{aligned} \tag{3}
$$

where $t_\nu : \mathbb{R} \to \mathbb{R}^+$ is the Student t-distribution function, t_ν^{-1} is the inverse function of t_ν and $t_{\rho,\nu}$ is the bivariate t-distribution, with parameters $\rho \in [-1,1]$ and $\nu \in \mathbb{R}^+$. Maximum likelihood provides us with estimates for parameters ρ and ν. The partial derivative of the bivariate t-copula is defined as

$$
\frac{\partial}{\partial u_2} C_{\rho,\nu}(u_1,u_2) = t_{\nu+1}\left(\frac{t_\nu^{-1}(u_1) - \rho t_\nu^{-1}(u_2)}{\sqrt{1-\rho^2}} \sqrt{\frac{\nu+1}{\nu + t_\nu^{-1}(u_2)^2}}\right). \tag{4}
$$

The partial derivative after u_1 follows in analogy. As stated in equation (1), the partial derivatives of the copula are simply the conditional distribution function $h_1(u_1|u_2)$ and $h_2(u_2|u_1)$. By calculating the $1-\alpha$- and α-percentile ($\alpha > 0$) of the conditional distribution function $h_2(u_2|u_1)$, we obtain the upper and lower confidence bands for U_2, conditional to U_1. Following the same approach, we are able to establish the upper and lower confidence bands of U_1, conditional to U_2. We set $\alpha = 0.05$ as ad hoc choice for all pairs, following Stander et al. (2013). In figure 1, an exemplary set of confidence bands based on the estimation period is depicted. We define three disjoint areas, A, B and C:

- A is defined as the intersection of the two areas above the upper confidence bands,

- B is defined as the intersection of the two areas under the lower confidence bands,

- C is defined as the unit square, minus areas A and B, i.e., $C = [0, 1]^2 \setminus \{A \cup B\}$.

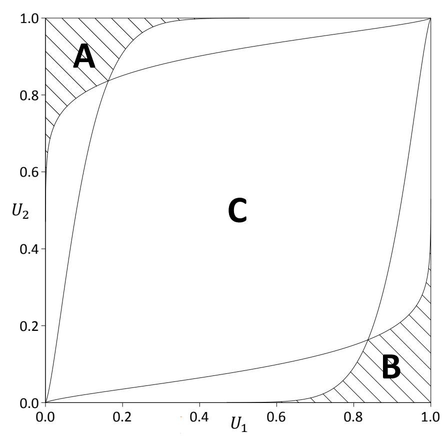

Figure 1: Confidence bands based on the estimation period and trading signals.

Copula function and confidence bands are estimated once using daily return time series in the estimation period and kept constant for the subsequent pseudo-trading period.

3.2.2 Pseudo-trading period: In the pseudo-trading period, all $n(n-1)/2$ pairs are traded according to the same set of rules. First, new incoming prices $p_{X_{1,t}}$ and $p_{X_{2,t}}$ $(t \in T_{ft})$ are used to calculate the log return realizations $r_{X_{1,t}}$ and $r_{X_{2,t}}$. Again, scaled ranks $u_{1,t}$ and $u_{2,t}$ are constructed, so we obtain $(u_{1,t}, u_{2,t}) \in [0, 1]^2$. Such points are exactly located in one of the sets A, B or C. Following Liew and Wu (2013) and Stander et al. (2013), we define corresponding trading entry signals:

- $(u_{1,t}, u_{2,t})$ is element of set A, i.e., stock 1 is undervalued and stock 2 overvalued. In consequence, we go long in stock 1 and short in stock 2.

- $(u_{1,t}, u_{2,t})$ is element of set B, i.e., stock 1 is overvalued and stock 2 undervalued. In consequence, we go short in stock 1 and long in stock 2.

- $(u_{1,t}, u_{2,t})$ is element of set C, i.e., both stocks are within their respective confidence bands. In consequence, we do not execute any trades.

We allow for pyramiding positions, i.e., we act upon every entry signal by investing one monetary unit, e.g., one USD per trade. For each pair i, we compute the cumulative return $CR_{ij,t}$ conditional to an entry signal j at time $t \in \{0, \ldots, 120\}$ for up to 120 days[1], i.e.,

$$CR_{ij,t} = log(PI_{ij,t}) - \underbrace{log(PI_{ij,0})}_{=0}, \quad t \in \{0, \ldots, 120\}, \tag{5}$$

where $PI_{ij,t}$ denotes a price index of pair i and entry signal j at time t, normalized to 1 at the time of the entry signal $t = 0$. Next, we aggregate the cumulative return for each pair i individually across all J entry signals to an aggregated cumulative return $ACR_{i,t}$:

$$ACR_{i,t} = \sum_{j=1}^{J} CR_{ij,t}, \quad t \in \{0, \ldots, 120\} \tag{6}$$

Figure 2 shows the aggregated cumulative return time series, conditional to the copula-based entry signals for one exemplary pair.

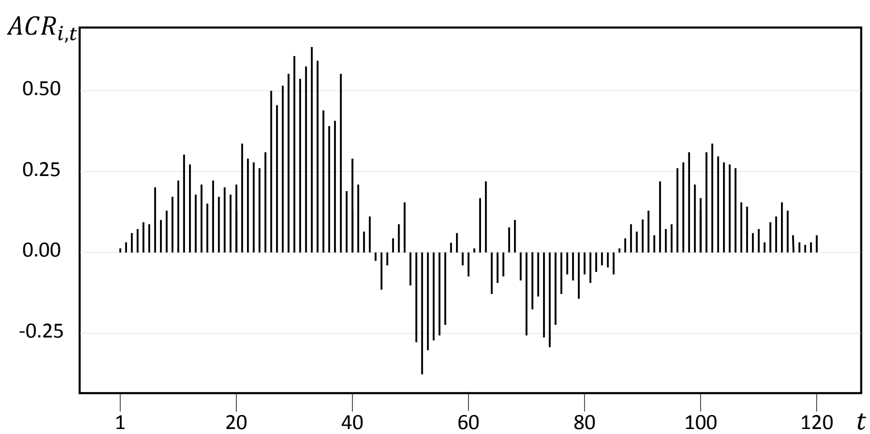

Figure 2: Conditional cumulative return time series of an example pair.

We use the aggregated cumulative return conditional to an entry signal ACR_{it} for each pair to (a) identify suitable pairs for the actual trading period and (b) to determine individualized per-pair exit thresholds.

[1]Strictly speaking, the 1 month pseudo-trading period is a 1 month signal-generation period. After each signal, we monitor the cumulative return for the subsequent 120 days.

Suitable pairs: First, we differentiate pairs in two types, mean-reversion pairs and momentum pairs. Mean-reversion pairs are profitable if traded according to the rules outlined above, since they tend to revert to their equilibrium-relationship after a copula-based entry signal. As such, the mean of the aggregated cumulative return times series $ACR_i = (ACR_{i,t})_{t \in 1,\ldots,120}$ is positive in expectation due to positive drift. Conversely, momentum pairs tend to diverge after the copula-based entry signals defined above, so the mean of the aggregated cumulative return time series ACR_i is negative in expectation, due to negative drift. For these pairs, trading rules have to be reversed in the actual trading period. For pairs selection, we sort all pairs in descending order based on their mean ACR_i. The top k pairs with highest mean ACR_i and minimum Spearman's ρ_{min} of 0.6 are retained as mean-reversion pairs. The bottom k pairs with lowest mean ACR_i and minimum Spearman's ρ_{min} of 0.6 are retained as momentum pairs. We set a flag for momentum pairs to reverse trading rules in the actual trading period in order to capture the momentum effect. The value of $\rho_{min} = 0.6$ is set ad hoc to introduce a minimum dependence between the two legs of a pair and to avoid trading of pairs without clear relationship. In case this condition is too rigid, we relax the correlation constraint until we have k top and k bottom pairs available[2].

Individualized exit rules: We determine individual take-profit and stop-loss levels for every pair i based on the cumulative return time series $CR_{ij} = (CR_{ij,t})_{t \in \{1,\ldots,120\}}$, one for each entry signal j. Specifically, for a mean-reversion pair, we set the average of the 95%-quantiles of the J cumulative return time series CR_{ij} as take-profit level and the average of the 10%-quantiles of the J cumulative return time series CR_{ij} as stop-loss level for the respective pair. In case we are dealing with a momentum pair, we reverse the trading rules, reconstruct the series CR_{ij} and proceed in analogy to mean-reversion pairs.

At the end of the formation period, we use the last 12 months of daily return data to re-estimate the t-copula and the respective confidence bands, following the procedure in subsection 3.2.1. This approach ensures that the parameters for the trading period are based on most recent estimates.

[2]This relaxation is introduced to ensure that we always have k pairs available for trading. However, relaxations rarely happen.

3.3 Trading period

Subsequent to the 60 month formation period T_f, consisting of 48 sets of 12 month estimation periods T_{fe} and 1 month pseudo-trading periods T_{ft} follows a 12 month trading period T_t. We only consider the top k pairs for trading, with $k \in \{5, 10, 20\}$. Trades are entered following the entry signals outlined in subsection 3.2.2. We only allow for one active position per pair, i.e., no pyramiding of positions in case of multiple entry signals. Trades are held until the take-profit or stop-loss levels are reached or until the end of the trading period. In line with Jegadeesh and Titman (1993) and Gatev et al. (2006), we run 12 overlapping trading periods in parallel, each with 12 months duration. We average across these 12 trading strategies starting one month apart to obtain one monthly return time series. Clearly, each trading period has its corresponding formation period, consisting of the prior 60 months. Following this approach, we use the first 60 months of the S&P 100 data, starting January 1990, exclusively for formation. After that, the first trading period begins. We log the returns, roll forward one month and repeat the process until the end of the sample period in December 2014 is reached. Hence, strategy returns are available from January 1995 until December 2014.

3.4 Return computation

We follow Gatev et al. (2006) and calculate the return on employed capital by dividing payoffs by the number of pairs that open during the trading period, i.e., we invest one USD for each active pair. Alternatively, we also show the return on committed capital, by scaling the payoffs by the number of pairs considered during the trading period, i.e., we have one USD available for each pair that may open - see Gatev et al. (2006) for further details.

4. Results

We run a fully-fledged performance evaluation, following Krauss et al. (2015a). We analyze the top 5, top 10 and top 20 mean-reversion and momentum pairs individually and compare them relative to a naive S&P 100 buy-and-hold strategy. Specifically, we examine the return distribution for each strategy variant, perform a value at risk analysis, evaluate risk-return characteristics, assess drawdown measures and check the exposure to common systematic risk factors with different factor models. Finally, we analyze the robustness of the strategies' returns to market frictions. The majority of selected performance metrics is discussed in Bacon (2008). All results in the subsequent tables encompass the trading part of the sample period from January 1995 to December 2014.

4.1 Return characteristics and trading statistics

In table 4, the return characteristics of the monthly mean-reversion and momentum pairs are depicted, relative to the S&P 100 buy-and-hold benchmark strategy. All strategy signals are implemented on the day of the signal, without delay. We see that the equal-weighted returns of the top 5 mean-reversion pairs are at 0.65 percent per month versus 0.59 percent for the top 5 momentum pairs. This level is approximately sustained for the top 10 pairs, but decreases to 0.38 percent for the top 20 mean-reversion pairs and to 0.49 for the top 20 momentum pairs. We may carefully conclude that our pairs selection algorithm outlined in section 3.2 is meaningful - a less rigorous selection of a higher number of pairs leads to lower returns. We observe a similar picture for the return on committed capital at a moderately reduced level, driven by the number of pairs that do not open during the trading period. The returns of all strategy variants are statistically and economically significant with Newey-West (NW) t-statistics above 5.8. The buy-and-hold strategy also leads to statistically significant monthly returns of 0.76 percent - well above the top 5 pairs, but at much higher levels of volatility. Particularly, the standard deviation of returns of the S&P 100 long-only benchmark is approximately 4.5 times higher than that of the pairs trading strategies, suggesting much higher risk. On the same note, the hit rate of the pairs-trading strategy is clearly better with approximately 74 percent of top 5 mean-reversion and 70 percent of top 5 momentum pairs exhibiting returns greater than zero, compared to 63 percent for the benchmark. The remainder of table 4 characterizes the return distributions. The top 5 pairs are positively skewed in both cases - a favorable property for investors. For the top 10 and top 20 pairs, we observe a mixed picture of positive and negative skewness. However, no strategy variant exceeds the negative skewness of the benchmark. High kurtosis suggests leptokurtic distributions for all strategy variants.

Table 5 reports the trading statistics. Of the top 5 mean-reversion pairs, 4.90 open during the trading period with an average number of 4.88 round-trips per pair. On average, these pairs are open for 2.83 months, indicating that we are dealing with a medium-term investment strategy.

	Mean-reversion			Momentum			Base
	Top 5	Top 10	Top 20	Top 5	Top 10	Top 20	S&P100
Value-weighted mean return	0.0055	0.0054	0.0045	0.0065	0.0059	0.0055	0.0069
Equal-weighted mean return	0.0065	0.0063	0.0038	0.0059	0.0054	0.0049	0.0076
Standard error (NW)	0.0007	0.0008	0.0006	0.0008	0.0008	0.0007	0.0031
t-Statistic (NW)	9.5046	8.0648	5.8542	6.9821	6.8399	6.5173	2.4540
Minimum	-0.0456	-0.0730	-0.0542	-0.0579	-0.0577	-0.0610	-0.1578
Quartile 1	0.0000	-0.0011	-0.0025	-0.0004	-0.0005	-0.0011	-0.0151
Median	0.0048	0.0045	0.0022	0.0040	0.0037	0.0031	0.0140
Quartile 3	0.0098	0.0111	0.0075	0.0107	0.0099	0.0103	0.0372
Maximum	0.0539	0.0451	0.0329	0.0700	0.0675	0.0574	0.1040
Share with return > 0	0.7375	0.6833	0.6250	0.6917	0.6792	0.6625	0.6292
Standard deviation	0.0099	0.0114	0.0095	0.0097	0.0093	0.0091	0.0449
Skewness	0.9073	-0.6208	-0.0748	0.5075	0.4200	-0.3186	-0.7704
Kurtosis	6.9292	10.1985	6.2489	14.4344	15.3365	14.0692	1.0651
Return on committed capital	0.0039	0.0041	0.0030	0.0022	0.0020	0.0018	0.0076

Table 4: Monthly return characteristics of top k mean-reversion and momentum pairs compared to a S&P 100 long-only benchmark.

Pairs portfolio	Top 5	Top 10	Top 20
A. Mean-reversion pairs			
Average number of pairs traded per 12 month period	4.90	9.78	19.20
Average number of round-trip trades per pair	4.88	5.16	5.15
Standard deviation of number of round trips per pair	0.70	0.53	0.40
Average time pairs are open in months	2.83	2.88	2.91
Standard deviation of time open, per pair, in months	0.83	0.70	0.62
B. Momentum pairs			
Average number of pairs traded per 12 month period	4.85	9.64	19.16
Average number of round-trip trades per pair	4.37	4.17	3.99
Standard deviation of number of round trips per pair	0.68	0.52	0.37
Average time pairs are open in months	2.68	2.71	2.89
Standard deviation of time open, per pair, in months	0.75	0.57	0.49

Table 5: Trading statistics for the top 5, top 10, and top 20 mean-reversion pairs (panel A) and momentum pairs (panel B).

4.2 Value at risk

Table 6 reports monthly value at risk (VaR) measures. Primarily, we opt for the historical VaR, as suggested by J.P. Morgan's RiskMetrics approach in Mina and Xiao (2001). In line with Huisman et al. (1999) and Favre and Galeano (2002), we additionally show the Cornish-Fisher (CF) VaR for more robust results if the higher moments are clearly different from those of a normal distribution. The tail risk of the mean-reversion and the momentum strategies is at a very low level compared to the benchmark, e.g., the historical VaR (1%) is -0.58 percent for the top 5 mean-reversion pairs versus -11.80 percent for the buy-and-hold strategy. For mean-reversion pairs, we see that tail risk measured by historical VaR (1%) slightly increases when including more pairs, i.e., from -0.58 percent (top 5) to -0.82 percent (top 20). This effect is surprising, as VaR-levels are expected to decrease with larger portfolios, since portfolio variance generally decreases with more securities. We conjecture that the top pairs selected with to the algorithm in section 3.2 have intrinsically lower risk involved. However, for momentum pairs, this effect is less pronounced. The better-suited Cornish-Fisher VaR produces higher levels of tail risk in light of the high kurtosis of the return distributions, but it still remains small in respect to the benchmark. The low risk of the pairs trading strategies is also expressed in maximum drawdown levels below 11 percent, compared to 56 percent for the benchmark. We summarize that the strategies exhibit favorable tail risk characteristics compared to a simple buy-and-hold alternative. Specifically, Gatev et al. (2006) conclude for a monthly historical VaR (1%) of -0.04320 for their top 5 pairs that a leverage ratio of 5:1 would be adequate. We achieve a very similar result for our top 5 pairs, suggesting that our current 2:1 leverage is a conservative choice for this strategy.

4.3 Annualized risk-return characteristics

In table 7, we discuss annualized risk-return ratios, providing the return investors obtain per unit of risk. All measures are based on equal-weighted returns. The Sharpe ratio is defined as excess return divided by standard deviation. We observe high values above 1.00 for all strategy variants (except top 20 mean-reversion) and - yet again - a moderate decline when including more pairs. All variants outperform the benchmark with a Sharpe ratio of 0.34. This finding is confirmed by the remaining risk-return measures. The Sortino ratio divides the returns by their downside deviation. Its main

	Mean-reversion			Momentum			Base
	Top 5	Top 10	Top 20	Top 5	Top 10	Top 20	S&P100
Historical VaR 1%	-0.0058	-0.0062	-0.0082	-0.0047	-0.0046	-0.0051	-0.1180
CF VaR 1%	-0.0227	-0.0510	-0.0327	-0.0446	-0.0462	-0.0482	-0.1231
Historical CVaR 1%	-0.0235	-0.0380	-0.0301	-0.0225	-0.0225	-0.0240	-0.1451
CF CVaR 1%	-0.0646	-0.0510	-0.0327	-0.0446	-0.0462	-0.0482	-0.1687
Historical VaR 5%	-0.0031	-0.0040	-0.0049	-0.0030	-0.0033	-0.0038	-0.0789
CF VaR 5%	-0.0056	-0.0121	-0.0108	-0.0058	-0.0059	-0.0084	-0.0744
Historical CVaR 5%	-0.0093	-0.0132	-0.0123	-0.0085	-0.0084	-0.0097	-0.1048
CF CVaR 5%	-0.0056	-0.0327	-0.0197	-0.0058	-0.0059	-0.0127	-0.1057
Minimum	-0.0456	-0.0730	-0.0542	-0.0579	-0.0577	-0.0610	-0.1578
Share with return ≤ 0	0.2625	0.3167	0.3750	0.3083	0.3208	0.3375	0.3708
Maximum drawdown	0.0693	0.1096	0.0946	0.0984	0.0846	0.0913	0.5562

Table 6: Monthly value-at-risk of top k mean-reversion and momentum pairs compared to a S&P 100 long-only benchmark.

advantage is the lower partial moment metric in the denominator, only measuring downside deviations as actual risk (compared to favorable upward deviations). We see that downside risk is even less expressed than "upside risk" for the copula based pairs trading strategies, leading to Sortino ratios above 2.9 for all variants versus 0.76 for the benchmark. Omega is a probability-weighted gain-loss ratio, relying on all higher moments. Again, all strategies exhibit strong outperformance relative to the benchmark.

	Mean-reversion			Momentum			Base
	Top 5	Top 10	Top 20	Top 5	Top 10	Top 20	S&P100
Return	0.0798	0.0773	0.0456	0.0722	0.0661	0.0594	0.0819
Excess return	0.0520	0.0496	0.0186	0.0445	0.0386	0.0321	0.0540
Standard deviation	0.0341	0.0396	0.0329	0.0335	0.0323	0.0317	0.1555
Downside deviation	0.0116	0.0188	0.0156	0.0135	0.0135	0.0145	0.1079
Sharpe ratio	1.5225	1.2511	0.5649	1.3299	1.1938	1.0116	0.3474
Sortino ratio	6.8483	4.1048	2.9176	5.3339	4.8866	4.0943	0.7584
Updside Potential ratio	7.3387	4.7586	4.0661	5.8547	5.4669	4.8014	2.3960
Omega	10.7999	6.3372	3.4657	9.0274	7.9918	6.1547	1.5448

Table 7: Annualized risk-return characteristics of top k mean-reversion and momentum pairs compared to a S&P 100 long-only benchmark.

4.4 Drawdown measures

Table 8 reports advanced drawdown metrics. Sterling and Calmar ratio both divide annualized return by the absolute value of maximum drawdown. For the Sterling ratio, the denominator is augmented by an additional 10 percent excess risk buffer. Considering the Calmar ratio, we see that the annual return of the top 5 and top 10 pairs is at more than 1.10 times the magnitude of maximum drawdown, compared to a value of 0.15 for the benchmark. In other words, these pairs trading strategies usually recover from a maximum drawdown in less than a year. Momentum strategies are slightly superior to mean-reversion strategies. The Burke ratio is defined as annualized excess return divided by the Euclidean norm of the drawdowns. This metric incorporates a total of d drawdowns, thus putting more weight on the larger ones. Nevertheless, we observe a similar picture as for Sterling and Calmar ratios. Pain index is the L1 norm of all drawdowns divided by the length of the series, providing a mean drawdown per observation. Along these lines, the Ulcer index is defined as "root-mean square measure of retracement" (Martin and McCann, 1989, p. 80), measuring the depth as well as duration of drawdowns. Across both metrics, the pairs trading strategies are well superior to the benchmark and momentum slightly superior to mean-reversion pairs. Scaling mean excess return by Pain or Ulcer index results in the Pain or Martin ratio, respectively, indicating even clearer outperformance of momentum relative to mean-reversion and of both strategies relative to naive buy-and-hold.

	Mean-reversion			Momentum			Base
	Top 5	Top 10	Top 20	Top 5	Top 10	Top 20	S&P100
Sterling ratio	0.4710	0.3687	0.2343	0.4569	0.4190	0.3686	0.1247
Calmar ratio	1.1505	0.7051	0.4819	1.2457	1.1452	0.9725	0.1472
Burke ratio	0.6312	0.3941	0.1534	0.5756	0.4946	0.3647	0.0679
Pain index	0.0030	0.0061	0.0069	0.0015	0.0016	0.0020	0.1751
Ulcer index	0.0119	0.0209	0.0194	0.0057	0.0059	0.0064	0.2528
Pain ratio	17.3942	8.1927	2.6781	30.2760	24.5958	16.3620	0.3084
Martin ratio	4.3514	2.3749	0.9579	7.7861	6.5803	4.9795	0.2136

Table 8: Drawdown measures of top k mean-reversion and momentum pairs compared to a S&P 100 long-only benchmark.

4.5 Subperiod analysis

Figure 3 summarizes the cumulative return of the top 5 mean-reversion and momentum pairs versus the S&P 100 benchmark in a single chart. One USD invested January 1995 grows to more than 4.5 USD for mean-reversion pairs and more than 4 USD for momentum pairs. Even though an investor achieves a similar result with the naive buy-and-hold strategy, its high volatility contrasts dramatically with the smooth growth of both pairs trading strategies without major drawdowns. When comparing figure 3 with figure 3 in Gatev et al. (2006), we observe that our strategy exhibits steady growth, whereas classical pairs trading shows decreasing returns.

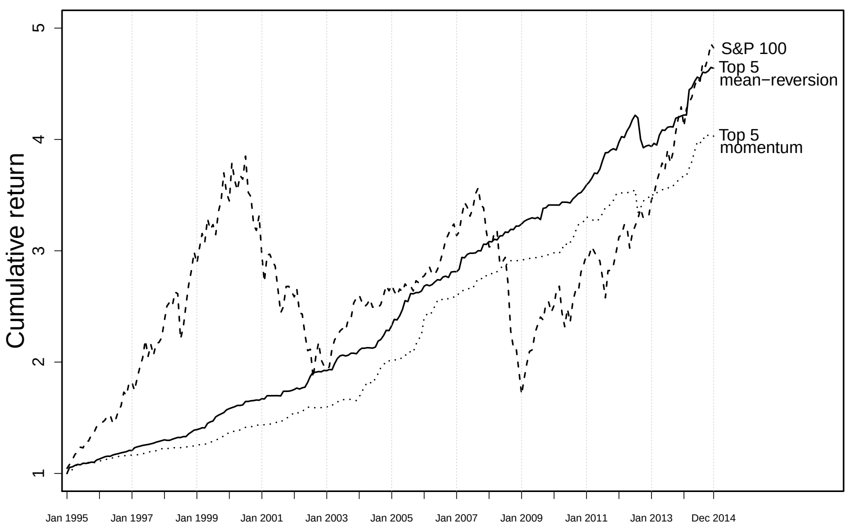

Figure 3: Cumulative return of top 5 mean-reversion pairs and top 5 momentum pairs versus S&P 100.

Figure 4 depicts the mean return per month of the top 5 mean-reversion and the top 5 momentum pairs versus the benchmark for the respective two-year period indicated on the x-axis. Again, the pairs trading strategies exhibit steady returns compared to the wild swings of the benchmark. Performance is very stable across time and does not seem to be impacted by crises, such as the burst of the dot-com bubble in 2000 or the financial crisis in 2008 with highly negative returns for the S&P 100. Also, strategy returns do not decline over time, as observed for classical pairs trading - see Gatev et al. (2006), figure 2. Most importantly, we do not observe a single two-year period with negative average returns for the top pairs.

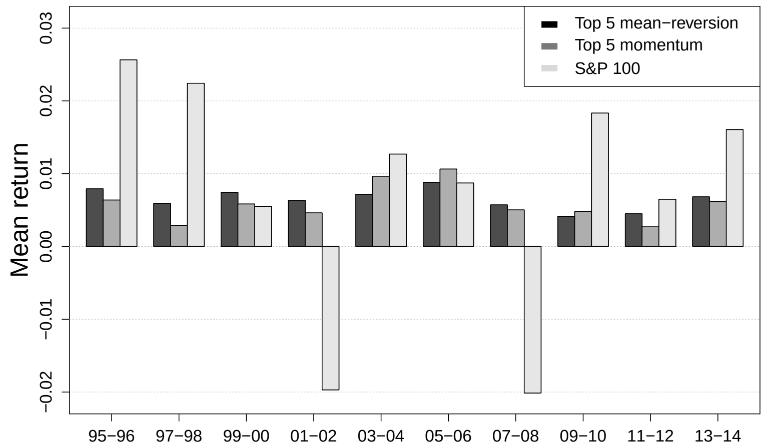

Figure 4: Mean return per month of top 5 mean-reversion pairs and top 5 momentum pairs versus S&P 100 for the respective period indicated on the x-axis.

4.6 Common risk factors

Finally, we are analyze the exposure of the strategies to common systematic sources of risk. In this respect, we perform three types of regressions. First, we deploy the standard Fama-French three-factor model (FF3), following Fama and French (1996). The latter captures exposure to the overall market, small minus big capitalization stocks (SMB) and high minus low book-to-market stocks (HML). Second, we enhance this model by a momentum factor and a short-term reversal factor along the lines of Gatev et al. (2006). We call this model Fama-French 3+2-factor model (FF3+2). Third, we use the recently developed Fama-French five-factor model, following Fama and French (2015). It consists of the three-factor model (FF5), enhanced by two additional factors, i.e., portfolios of stocks with robust minus weak profitability (RMW) and with conservative minus aggressive (CMA) investment behavior. All data related to these models is downloaded from Kenneth R. French's website[3]. Findings for the top 5 mean-reversion and momentum pairs are

[3] We thank Kenneth R. French for providing all relevant data for these models on his website.

summarized in table 9. Standard errors are depicted in parentheses.

	Mean-reversion top 5			Momentum top 5		
	FF3	FF3+2	FF5	FF3	FF3+2	FF5
(Intercept)	0.0073**	0.0064**	0.0078**	0.0061***	0.0058**	0.0063**
	(0.0011)	(0.0019)	(0.0021)	(0.0013)	(0.0011)	(0.0014)
Market	0.2146*	0.2089*	0.2215*	0.1590*	0.1214	0.1530*
	(0.0963)	(0.1074)	(0.1165)	(0.0864)	(0.0564)	(0.0813)
SMB	0.2034*	0.1896*		0.1762*	0.1630*	
	(0.0873)	(0.0769)		(0.0601)	(0.0689)	
HML	−0.0861	−0.0947*		−0.1163*	−0.1289*	
	(0.0432)	(0.0579)		(0.0891)	(0.0922)	
Momentum		0.1823			0.4321*	
		(0.0872)			(0.0631)	
Reversal		0.3314*			0.0689	
		(0.1347)			(0.0697)	
SMB5			0.1942*			0.1221*
			(0.0741)			(0.0632)
HML5			−0.0712			−0.1121*
			(0.0449)			(0.0641)
RMW5			1.2134			0.8641
			(0.7242)			(0.6421)
CMA5			−0.2541			−0.1877
			(0.1544)			(0.1134)
R^2	0.2743	0.2921	0.2841	0.1942	0.2021	0.2011
Adj. R^2	0.2611	0.2864	0.2654	0.1843	0.1921	0.1899
Num. obs.	240	240	240	240	240	240
RMSE	0.0921	0.0901	0.0928	0.0798	0.0814	0.0803

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Table 9: Exposure to systematic sources of risk earnings momentum strategies.

Irrespective of the factor model employed, we see that the returns of the top 5 mean-reversion and momentum pairs have statistically and economically significant alphas ranging between 0.58 and 0.78 percent per month - slightly higher than the raw returns. Even though the strategy is dollar-neutral, all factor models indicate a marginal but statistical significant positive loading on the market. Also, we observe slight yet significant exposure to the SMB factor. The latter result is surprising at first sight, given that we exclusively invest in a large capitalization stock universe. However, other studies observe comparable aberrations, see Chan et al. (2002) for a similar anomaly in respect to large cap mutual funds. Chen and Bassett (2014) provide an explanation, effectively showing that the Fama-French models may attribute small size to large cap stocks and portfolios, due to the coexistence of the market factor with the SMB factor in the regressions.

More interesting are the coefficients on the momentum and the reversal factor. For mean-reversion pairs, we find a statistical significant loading on the reversal factor, indicating that we short short-term winners and buy short-term losers. Conversely, the momentum factor is not significant. For the momentum pairs, the situation is exactly reversed, indicating that we buy short-term winners and short short-term losers. The recent Fama-French five factor model does not offer any additional explanatory power, with statistically insignificant RMW and CMA factors.

4.7 Market frictions

In this subsection, we evaluate the robustness of the pairs trading strategies in light of market frictions. Most notably, we consider a one-day-waiting rule and estimates for trading costs in a high-liquidity stock universe. Table 10 depicts the monthly equal-weighted (EW) returns of the strategies without and with a one-day-waiting rule, as indicated by one asterisk. We find that delayed execution only has a minor negative impact of 2 or 3 basis points (bps) on the top 5 pairs. The EW returns of the top 20 pairs are reduced by 6 bps (mean-reversion) and 8 bps (momentum), respectively. However, we see that returns remain statistically and economically significant, even with delayed execution.

Trading costs for pairs trading strategies have been analyzed by Do and Faff (2012). For the recent years of their sample, i.e., 1995-2009, they estimate institutional commissions to decline from 16 bps to 7-9 bps per transaction. Bogomolov (2013) finds that even retail commissions are at approximately 10 bps per transaction. Given that our sample extends until 2015 and trading costs have become even lower with decimalization, the rise of dark pools and rebates for liquidity provision, we conservatively estimate average institutional commissions at 10 bps per transaction. Do and Faff (2012) set the market impact for the entire CRSP stock universe for the same period at approximately 20 bps. This estimate is too high for a highly liquid universe of blue chip stocks. Krauss et al. (2015b) use 1-min binned high-frequency data and determine an average bid-ask spread of 4-5 bps for the 30 most liquid German stocks in the DAX 30 in 2014. We assume a value of 5 bps as proxy for the S&P 100 constituents as well. Short-selling costs are negligible. Hence, we have round-trip trading costs for each leg of a pair of $2 \cdot 10 + 5 = 25$ bps, i.e., two-times commission and one-time crossing of the bid-ask spread. For the entire pair, round-trip trading costs are thus 50 bps. Table 10 also depicts estimates for strategy profits after one-day-waiting and transaction costs, indicated by two asterisks. Even in this very conservative scenario, return estimates after consideration of market frictions remain statistically significant, except for the top 20 mean-reversion pairs. The top 5 pairs still achieve more than 0.40 percent per month. We conclude that the strategies are feasible in light of market frictions, posing a severe challenge to the semi-strong form of market efficiency.

	Mean-reversion			Momentum		
	Top 5	Top 10	Top 20	Top 5	Top 10	Top 20
Average number of round-trip trades per pair per month	0.4064	0.4301	0.4289	0.3645	0.3473	0.3321
EW mean return	0.0065	0.0063	0.0038	0.0059	0.0054	0.0049
Standard error	0.0007	0.0008	0.0006	0.0008	0.0008	0.0007
t-Statistic (NW)	9.5046	8.0648	5.8542	6.9821	6.8399	6.5173
EW mean return*	0.0062	0.0055	0.0032	0.0061	0.0056	0.0041
Standard error (NW)*	0.0009	0.0010	0.0008	0.0009	0.0009	0.0008
t-Statistic (NW)*	7.0513	5.7664	4.0365	6.4977	6.4322	5.4461
EW mean return**	0.0042	0.0034	0.0010	0.0043	0.0038	0.0025
Standard error (NW)**	0.0009	0.0010	0.0008	0.0009	0.0009	0.0008
t-Statistic (NW)**	4.7424	3.5250	1.3076	4.5689	4.4286	3.2593

Table 10: Impact of one-day-waiting rule (one asterisk) and one-day-waiting rule plus trading costs (two asterisks) on monthly equal-weighted (EW) mean return of top k mean-reversion and momentum pairs.

5. Conclusion

In this paper, we develop an integrated copula-based pairs trading framework and apply it to the constituents of the S&P 100 index.

Our first contribution is purely methodological. To our knowledge, we are the first authors using copulas along the entire pairs trading value chain, i.e., for pairs formation and trading. Specifically, we select stocks based on pseudo-trading profitability and a minimum Spearman's ρ, thereby sifting out spurious relationships. Also, we rigorously differentiate between mean-reversion and momentum pairs - a further novelty to pairs trading literature. Finally, we implement individualized quantile-based trading rules for each pair. Contrary to the existing return-based copula approaches, we also consider the time structure of returns by centering pairs selection on the cumulative return time series, conditional to copula-based entry signals.

Our second contribution is purely empirical. We apply the pairs trading framework to the highly liquid S&P 100 constituents and run an in-depth performance evaluation. We find high and statistically significant raw returns of 7.98 percent per year for the top 5 mean-reversion and of 7.22 percent per year for the top 5 momentum pairs. These results are largely robust when considering a one-day-waiting rule and trading costs of 0.5 bps per round-trip trade per pair. Also, the returns cannot be attributed to common sources of systematic risk, leaving us with statistically and economically significant monthly alphas. Highly appealing are the risk characteristics of the pairs trading strategy: Monthly standard deviation for the top 5 mean-reversion and momentum pairs is only at approximately 20 percent of the monthly standard deviation of a naive S&P 100 buy-and-hold strategy. When considering downside deviation, this factor even decreases to approximately 10 percent, meaning that the downside risk is significantly reduced relative to the long-only benchmark. This finding is consistent across various measures of tail risk, i.e., value-at-risk and drawdown metrics. Specifically, the attractive VaR levels would suggest that an even higher leverage ratio than 2:1 could be adequate for this low-risk strategy.

Compared to the existing pairs trading literature, our findings exhibit the following properties. First, we find statistically and economically significant returns on a highly liquid stock universe from 1995 to 2014. Conversely, Gatev et al. (2006) and later Do and Faff (2010) show declining profitability for classical distance-based pairs trading. We carefully conjecture that per-pair individualized trading rules and the consideration of nonlinear dependencies are more likely to mimic the behavior of proprietary trading desks and hedge funds, thus explaining sustained profitability levels versus classical pairs trading. The results are particularly convincing, since our application is focused on a very efficient large cap market segment and on times when sufficient computational resources were available for sophisticated strategies. As such, we believe that - contrary to generalized pairs trading algorithms - individualized trading rules are less likely to be arbitraged away.

In light of these findings, we conclude that our variant of copula-based pairs trading poses a severe challenge to the semi-strong form of market efficiency. In particular, we show the potential of refined statistical arbitrage strategies even in seemingly efficient market segments and in modern times.

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