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Non-Extensitivity versus Informative Moments for Financial Models - A Unifying Framework and Empirical Results

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SUMMARY

Information-theoretic approaches still play a minor role in financial market analysis. Nonetheless, there have been two very similar approaches evolving during the last years, one in so-called econophysics and the other in econometrics. Both generalize the notion of GARCH processes in an information-theoretic sense and are able to capture skewness and kurtosis better than traditional models. In this article we present both approaches in a more general framework and compare their performance in some illustrative data sets.

Keywords and phrases: Entropy density; Skewness; Kurtosis, GARCH
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1 Introduction

Though information-theoretic approaches still play only a minor role in financial market analysis, there have been two recent approaches developing independently one in econophysics and the other in econometrics. Both approaches generalize the notion of either ARCH or GARCH models.

The econometric models for financial markets analysis have been introduced by Rockinger/ Jondeau (2002) and been extended and generalized in Bera/Park (2009) and Fischer/Herrmann (2008). These approaches model variance's motion in time, like traditional GARCH models, but include knowlegde of higher moments, such as skewness and kurtosis, in order to capture known stylized facts.

The models in econophysics have been developed mainly in Queirós/Tsallis (2005), Borland (2005) and Queirós (2007). The econophysics approaches model only variance, but derive corresponding maximum entropy distributions by maximizing Tsallis q-entropy

$$H_T = \int_{D(X)} \frac{1 - f(X)^q}{q-1} dx,$$

where X is some random variable, $D(X)$ its support and $f(x)$ its density function, instead of the Shannon or Boltzmann-Gibbs entropy for given constraints. The resulting distribution

given only variance constraints is called q-Gaussian and its functional form is equivalent to a generalized Student-t distribution. A deviation of the value found for q from 1, the case when the normal distribution would be recovered, is called non-extensitivity.
Both approaches have been succesfully applied to financial market data as both generalizations allow to capture kurtosis in given variance models. Following Kesavan/Kapur (1989) we give a more general framework for information theoretic models for time-varying moments and derive GARCH models as well as the above mentioned non-extensive approaches as special cases. We extend the existing models and in an application to financial market data we compare these model's capability to capture financial returns behavior.

2 Models for Time-Varying Moments using the Generalized Principle of Maximum Entropy

In this note we will only consider time-series models that assume the conditional distribution of some random variable X at time t given the information set available at time $t-1$ to be the distribution maximizing some generalized entropy measure as

$$H(f) = -\int_{D(X)} \phi(f(x))dx,$$

for some convex function[1] ϕ and for $D(X)$ the random variables support, subject to constraints of $k+1$ conditional expectation values of suitable functions $g_i(\cdot)$ as

$$E(g_0(X)) = E(1) = 1, \quad E\left(g_i(X)|\mathcal{F}_{t-1}\right) = m_i(\mathcal{F}_{t-1}), \quad \forall i = 1, ..., k,$$

where a_i may be some deterministic functions depending only on the information set available.

The information theoretic interpretation of such models is, that we model only some expectation values' motion in time and for all information missing to completely determine the corresponding density functions, we maximize entropy. In the case of the Shannon entropy measure such a way of modelling is nothing else but the consequent application of Jaynes (1957) principle of maximum entropy. Using generalized entropy measures is justified by Kesavan/Kapur (1989)'s generalized maximum entropy principle.

Following their suggestion, we restrict ϕ to the set of differentiable convex functions. A variational approach shows that, under some weak conditions for ϕ, for the maximum entropy density (if it exists) holds that,

$$\phi'(f(x)) = \sum_{i=0}^{k} \lambda_i g_i(x), \tag{2.1}$$

[1] Here we follow the suggestion of Kapur/Kesavan (1989).

where the λ_i have to be chosen such that the constraints are fullfilled.[2] If a solution exists for f, the λ_i can be found by numerically minimizing a concave function of some dual problem. Some generalized entropy measures and corresponding dual problems are given e.g. in Kapur (1994). Table (1) gives some examples:

Entropy	$\phi(t)$	Parameters
Burg	$ln(t)$	-
Kapur	$t \cdot ln(t) + \frac{1}{c}(t+c)ln(t+c)$	$c > 0$
Shannon	$t \cdot ln(t)$	-
Tsallis	$\frac{1-t^q}{1-q}$	$q > 0,\ q \neq 1$

Table 1: Some generalized entropy measures.

A very efficient numerical algorithm for their implementation is given by Rockinger/Jondeau (2002) for the Shannon entropy, but can easily be adjusted to other dual problems.

3 GARCH Models as Models for Time-Varying Moments

Bollerslev (1986)'s original GARCH(p,q) model may be given as

$$x_t|\mathfrak{F}_{t-1} = z_t \cdot \sigma_t, \quad z_t \overset{iid}{\sim} \mathcal{N}(0,1),$$
$$\sigma_t = \alpha_0 + \sum_{i=1}^{p} \alpha_{1,i} x_{t-i}^2 + \sum_{i=1}^{q} \alpha_{2,i} \sigma_{t-i}^2, \tag{3.1}$$

where x_t is the random variable at time t and $\mathfrak{F}_{t-1}$ the information set available in the period before.

We can present the same model as a model for time-varying moments as defined above, if we derive the conditional densities of x_t given the information available at $t-1$, by maximizing Shannon's entropy measure

$$H_S(f) = \int_{D(X)} f(x) ln(f(x)) dx$$

subject to the constraints

$$E(1|\mathcal{F}_{t-1}) = 1,\ E(X|\mathcal{F}_{t-1}) = \mu,\ E((X-\mu)^2|\mathcal{F}_{t-1}) = \sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_{1,i} x_{t-i}^2 + \sum_{i=1}^{q} \alpha_{2,i} \sigma_{t-i}^2.$$

We can derive the Shannon entropy as a generalized measure of entropy, if we set $\phi(t) = t \cdot ln(t)$. Because

$$\phi'(t) = \frac{\delta(t \cdot ln(t))}{\delta t} = 1 + ln(t),$$

[2] Compare e.g. Kapur(1994).

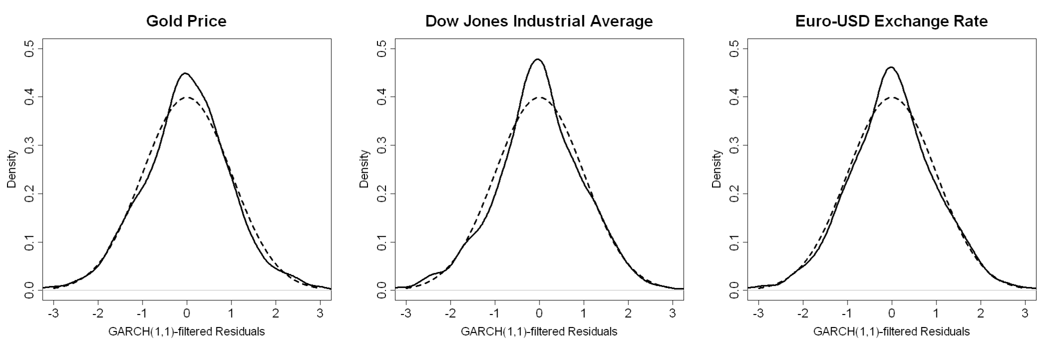
we derive the funtional form of the conditional distribution using equation (2.1) as

$$f(x|\mathfrak{F}_{t-1}) = exp(-1 + \lambda_0 + \lambda_1 x + \lambda_2 x^2),$$

where the λ_i are functions of the conditional moment constraints. As we derive λ to meet the above constraints, this distribution is nothing else but the normal distribution with mean μ and variance σ_t^2, just as in Bollerslev (1986)'s model.

It is a well known fact, that GARCH-models assuming gaussian innovations do not sufficiently describe financial markets data. Apart from a vast literature concerned with that topic[3], this can be seen be looking at the empirical distribution of GARCH-filtered innovations. After estimating $\alpha_0, \alpha_{1,1}, ..., \alpha_{1,p}, \alpha_{2,1}, ..., \alpha_{2,q}$, these can be obtained by rewriting GARCH-models in the original notation of Bollerslev (1986), as $\hat{z}_t = \frac{x_t}{\sigma_t}$

If the assumption of gaussian innovations would hold, the empirical distribution of $\hat{z}_t$ should be close to a normal distributions. But having a look at some illustrative data sets[4], we find some evidence, that this assumption does not hold, see figure (1).



	Gold		DJIA		EurUS	
	Statistic	p-Value	Statistic	p-Value	Statistic	p-Value
Skewness	-0.0309		-0.0503		-0.0404	
Kurtosis	3.8532		3.7080		3.8534	
Jarque-Bera	49.514	1.771e-11	35.124	3.064e-08	50.514	1.074e-11
χ^2	11.382	0.2504	34.602	5.666e-05	16.084	0.0652

Figure 1: GARCH(1,1) innovations' empirical densities versus normal density (dotted line) and results for tests on normality.

It is generally assumed that financial market GARCH innovations deviate from the normal distribution because of skewness and higher kurtosis. Traditional approaches try to overcome that problem by assuming z_t to follow some parametric distribution that is flexible enough

[3]See e.g. Hansen (1994) for an overview.

[4]See section (6.2) for a more detailed description of the data sets.

to model skewness and kurtosis, such as e.g. the SGT2, EGB2 or the family of stable distributions.[5][6]

This note, of course, is devoted to information theoretic approaches that directly model such features by, e.g. including measures of skewness and kurtosis in the entropy maximization task or, as the non-extensive approaches do, by choosing suitable entropy measures to allow for higher kurtosis.

4 Higher Informative Moments

To the best of our knowlegde, the first approach of including knowledge of higher moments in an information theoretic framework was given by Rockinger/Jondeau (2002). In their approach the use third and fourth power moments to measure skewness and kurtosis and use Shannon's entropy as information measure. But as Fischer/Herrmann (2008) point out, these measures do not allow the derivation of proper maximum Shannon entropy densities for kurtosis values higher than implied by the lower moments. In order to overcome that problem, one has to use measures that are defined as expectation values of functions that grow slower in x than x^2.[7] Suggestions for such measures as $m_i = E(g_i(X)|\mathfrak{F}_{t-1})$ can be found in Bera/Park (2009) and Fischer/Herrmann (2008), as e.g.

m_3	m_4
$E\left(tan^{-1}(X)\right)$	$E\left(tan^{-1}(X)^2\right)$
$E\left(\frac{X}{1+X^2}\right)$	$E\left(log(1+X^2)\right)$

Table 2: Some suggestions for measures of asymmetry m_3 and measures of kurtosis m_4 as proposed in recent literature.

where m_3 denotes measures of asymmetry and m_4 denotes measures of kurtosis.

Following Fischer/Herrmann (2008) and Bera/Park (2009), we can extend Bollerslev GARCH model as model for time-varying moments, assuming for the conditional distribtions of X_t given the information set available at $t-1$ the distribution maximizing the Shannon entropy subject to

$$E\left(X|\mathfrak{F}_{t-1}\right) = \mu, \quad E\left((X-\mu)^2|\mathfrak{F}_{t-1}\right) = \sigma_t,$$

$$E\left(g_3\left(\frac{X-\mu}{\sigma_t}\right)|\mathfrak{F}_{t-1}\right) = m_3, \quad E\left(g_4\left(\frac{X-\mu}{\sigma_t}\right)|\mathfrak{F}_{t-1}\right) = m_4,$$

where we let σ_t vary over time as in the original GARCH model, see equation (3.1), while we assume all other moment functions to be constant over time.[8]

[5] Compare e.g. Theodossiou (1998), MacDonald (1991) and Rachev/Mittnik (2000).

[6] Some of the generalized parametric approaches may also find some representation in the above framework - but because of the vast supply of such suggestions, we will restrict this note to the most basic case.

[7] Compare Fischer/Herrmann (2008).

[8] Nonetheless there is of course weak evidence of time-variability of higher moments, compare Fischer/Herrmann(2008) or e.g. Dubauskas/Teresiene(2005).

5 Non-Extensive Approaches

Information theoretic models in econophysics have been developed mainly in Queirós/Tsallis (2005), Borland (2005) and Queirós (2007). Their approaches generalize GARCH models in the above discussed form by assuming the conditional density functions to maximize the generalized entropy measure suggested by Tsallis (1988)[9]

$$H_T = \int_{D(X)} \frac{1 - f(X)^q}{q-1} dx,, \quad \text{s.t. } E\left(X|\mathfrak{F}_{t-1}\right) = \mu, \quad E\left((X-\mu)^2|\mathfrak{F}_{t-1}\right) = \sigma_t, \tag{5.1}$$

with different suggestions for σ_t's motion in time.

Contrary to the econometric approaches these models capture kurtosis not as information explicitly included, but by flexibly varying q. As the resulting conditional distributions of these models are

$$f_{ME,T} = \left(\frac{q-1}{q}\sum_{i=0}^{k}\lambda_i g_i(x)\right)^{\frac{1}{q-1}} = \left(\frac{q-1}{q}\left(\lambda_0 + \lambda_1 x + \lambda_2 (x-\mu)^2\right)\right)^{\frac{1}{q-1}},$$

they can be interpreted as some generalized t-distribution, where q is a parameter that drives the degrees of freedom, ν. For fixed mean and variance a finite ν corresponds to a distribution with higher kurtosis than the normal distribution. Because distributions with a value for q different than 1 are called non-extensive[10], we refer to these approaches as non-extensive approaches. As the degree of this non-extensitivity increases with the kurtosis implied, these models capture kurtosis by their non-extensitivity.

6 Application to Financial Market Data

We compare three models, equivalent in their flexibility, by applying them to three time series typical for financial market data from three different markets. We will compare these models by likelihood and likelihood based goodness-of-fit measures, by the distance of their empirical innovations' distribution to the theoretical model and by their capability to explain their empirical quantiles in the tails.

6.1 Models

For the class of the econometric models (ECO) we follow a suggestion by Bera/Park (2009) and maximize the Shannon entropy subject to

$$E\left(X|\mathfrak{F}_{t-1}\right) = \mu, \quad E\left((X-\mu)^2|\mathfrak{F}_{t-1}\right) = \sigma_t, \tag{6.1}$$

[9] Tsallis suggestion is also known as the entropy of Havrda/Charvat.

[10] Compare e.g. Tsallis (1988).

and include knowledge of higher moments using

$$E\left(tan^{-1}\left(\frac{X-\mu}{\sigma_t}\right)|\mathfrak{F}_{t-1}\right)=m_3,\quad E\left(ln\left(1+\left(\frac{X-\mu}{\sigma_t}\right)^2\right)|\mathfrak{F}_{t-1}\right)=m_4, \tag{6.2}$$

as in some previous study models using these moments performed best.

For the class of non-extensive models (NEX) we maximize the Tsallis entropy given in equation (5.1) subject to equation (6.1) and, contrary to the proposal in econophysicy, additionally introduce the skewness moment of equation (6.2). The later is included to make sure, that the non-extensive models exhibit the same flexibility as the econometric models in terms of skewness.

In order to show the flexibility of the proposed generalized framework, we suggest a third model (KAP), where we maximize the entropy measure suggested by Kapur (1994), subject to equation (6.1) and the skewness moment of equation (6.2), where the flexibility in terms of kurtosis shall be introduced by flexibly varying c.

For all models we assume the time-model for variance as in a traditional GARCH(1,1)[11] as

$$\sigma_t=\alpha_0+\alpha_1 x_{t-i}^2+\alpha_2\sigma_{t-i}^2$$

and assume all higher moments or parameters to be constant over time.

6.2 Data

In order to sample different kinds of financial market indices, we chose the daily returns between January 1st 2003 and March 20th 2009 for the gold price, the Dow Jones Industrial Average, both from *yahoo.finance.com*, and for the euro-US-dollar exchange rate from *www.ecb.int*.

Some descriptive statistics for the data are given in table (3):[12]

	Gold	DJIA	EurUS
Mean	-1.363e-03	2.316e-05	-1.657e-04
Stan.Dev.	3.479e-02	1.286e-02	6.499e-03
$\hat{m_3}$	-1.596e-01	-5.802e-02	7.545e-02
$\hat{m_4}$	6.961	14.301	7.254
Observations	1606	1603	1632

Table 3: Some descriptive statistics for the illustrative data sets.

[11] As there is empirical evidence that $p=1$ and $q=1$ sufficiently describe financial returns behavior, compare e.g. Bera/Higgins (1993).

[12] m_3 (m_4) denotes the third (fourth) standardized power moment.

6.3 Empirical Results

We use numerical optimization routines to implement a maximum likelihood estimation of the model parameters. Estimates, log-likelihood and standard errors (in brackets) are given in figure (2):

Gold Price

	$\widehat{\alpha_0}$	$\widehat{\alpha_1}$	$\widehat{\alpha_2}$	$\widehat{\beta_0}$	$\widehat{\gamma_0}$	LogL
GARCH	7.2937e-06 (3.5009e-06)	0.04064 (0.00753)	0.95317 (0.00852)	-	-	3268.252
ECO	7.6282e-06 (4.1548e-06)	0.04101 (0.00874)	0.95246 (0.00991)	0.00409 (0.00393)	0.50789 (0.00534)	3283.845
NEX	7.028e-06 (3.9615e-06)	0.03936 (0.00824)	0.95468 (0.00922)	0.00537 (0.00385)	0.80672 (0.03794)	3284.064
KAP	7.6789e-06 (4.1579e-06)	0.04158 (0.00886)	0.95197 (0.01)	0.0052 (0.00375)	3.88308 (1.01096)	3283.851

Dow Jones Industrial Average

	$\widehat{\alpha_0}$	$\widehat{\alpha_1}$	$\widehat{\alpha_2}$	$\widehat{\beta_0}$	$\widehat{\gamma_0}$	LogL
GARCH	7.0443e-07 (2.7215e-07)	0.07095 (0.01010)	0.92296 (0.01025)	-	-	5241.281
ECO	7.0784e-07 (3.1362e-07)	0.07065 (0.01147)	0.92299 (0.01158)	-0.00004 (0.00399)	0.50851 (0.00531)	5255.646
NEX	7.3838e-07 (3.1597e-07)	0.07024 (0.01143)	0.92337 (0.01149)	-0.00114 (0.00393)	0.81873 (0.04088)	5252.580
KAP	7.3096e-07 (3.1737e-07)	0.07018 (0.01148)	0.92328 (0.01163)	-0.00284 (0.00384)	4.0709 (1.08256)	5255.698

Euro-USD Exchange Rate

	$\widehat{\alpha_0}$	$\widehat{\alpha_1}$	$\widehat{\alpha_2}$	$\widehat{\beta_0}$	$\widehat{\gamma_0}$	LogL
GARCH	1.1797e-07 (7.5857e-08)	0.03380 (0.00584)	0.96342 (0.00592)	-	-	6062.748
ECO	1.4706e-07 (9.2357e-08)	0.03371 (0.00675)	0.96254 (0.00702)	-0.00144 (0.00386)	0.50931 (0.00524)	6076.519
NEX	1.6122e-07 (9.0189e-08)	0.03127 (0.00638)	0.96441 (0.00674)	-0.00272 (0.00379)	0.81997 (0.03741)	6076.861
KAP	1.4455e-07 (9.2727e-08)	0.03434 (0.00682)	0.96211 (0.00706)	-0.00236 (0.00369)	3.63563 (0.99112)	6075.98

Figure 2: Empirical results for our models for some illustrative data sets.

We find for all models similar estimates for variance motion in time as well as similar values for skewness. The kurtosis parameters differ of course, as all models have their own way to capture kurtosis. For all models and all data sets we find significant non-normal kurtosis.[13]

[13] The value for m_4 (as used in the "ECO" model) implied by the normal distribution is about 0.53345.

Figure (3) gives an overview over the model's fit, where LogL denotes log-likelihood, AIC the Akaike information criterion, BIC the Bayesian information criterion, KS the Kolgomorov-Smirnov distance and χ^2 the χ^2-test statistic with 10 classes.

Gold Price

	LogL	AIC	BIC	KS	χ^2
GARCH	3268.252	-6530.503	-6514.359	1.3004	19.560
ECO	3283.845	-6557.690	-6530.782	0.5238	4.666
NEX	3284.064	-6558.129	-6531.221	0.5626	4.396
KAP	3283.851	-6557.703	-6530.795	0.4707	4.037

Dow Jones Industrial Average

	LogL	AIC	BIC	KS	χ^2
GARCH	5241.281	-10476.56	-10460.42	2.0280	42.770
ECO	5255.646	-10501.29	-10474.39	1.6563	23.776
NEX	5252.58	-10495.16	-10468.26	1.7200	29.956
KAP	5255.698	-10501.40	-10474.50	1.4537	21.874

Euro-USD Exchange Rate

	LogL	AIC	BIC	KS	χ^2
GARCH	6062.748	-12119.50	-12103.30	1.2970	21.442
ECO	6076.47	-12142.94	-12116.05	0.6817	5.747
NEX	6076.811	-12143.62	-12116.73	0.7017	9.754
KAP	6075.93	-12141.86	-12114.97	0.6125	5.217

Figure 3: Some goodness-of-fit measures for the applied models.

All generalizations of the GARCH model behave well - except for the Dow Jones Industrial Average, where for the χ^2-test would reject all suggested models. In the sense of likelihood-based measures we can not derive a uniformly "best" model. The Kapur model outperforms all other models if we use only KS and χ^2 as criteria.

We conclude that the inclusion of knowledge of higher moments such as skewness and kurtosis significantly improves our model - not only in the sense of likelihood, but also in likelihood based measure that penalize additional parameters as AIC or BIC and also in the sense of distance-based measures of fit, such as χ^2 or KS.

Comparing the generalized models with each other, we find very similar results. This is of course due to the fact, that all models basically rely on the same additional information and only differ in the way of measuring this information.

7 Summary

The information-theoretic approaches to time series models considered in econometrics and econophysics may both be interpreted as special cases of models for time-varying moments using the generalized maximum entropy principle. Using this interpretation we derive a third model. Using three illustrative data sets typical for financial markets, we show that all suggested models from this class exhibit similar flexibility in capturing financial returns behavior.

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