IWQW

Institut für Wirtschaftspolitik und Quantitative
Wirtschaftsforschung

Diskussionspapier
Discussion Papers

No. 3/2010

Volatility Models with Innovations from New Maximum Entropy Densities at Work

Matthias Fischer
University of Erlangen-Nuremberg
Yang Gao
University of Erlangen-Nuremberg
Klaus Herrmann
University of Erlangen-Nuremberg

ISSN 1867-6707

Friedrich-Alexander-Universität
IWQW
Institut für Wirtschaftspolitik und Quantitative Wirtschaftsforschung

VOLATILITY MODELS WITH INNOVATIONS FROM NEW MAXIMUM ENTROPY DENSITIES AT WORK

MATTHIAS FISCHER, YANG GAO AND KLAUS HERRMANN

Department of Statistics and Econometrics
University of Erlangen-Nürnberg, Germany

Email: Matthias.Fischer@wiso.uni-erlangen.de

SUMMARY

Generalized autoregressive conditional heteroskedasticity (GARCH) processes have become very popular as models for financial return data because they are able to capture volatility clustering as well as leptokurtic unconditional distributions which result from the assumption of conditionally normal error distributions. In contrast, Bollerslev (1987) and several follow-ups provided evidence that starting with leptokurtic and possibly skewed (conditional) error distributions will achieve better results. Parallel to these flexible but to some extend arbitrary chosen parametric distributions, recent years saw a rise in suggestions for maximum entropy distributions (e.g. Rockinger and Jondeau, 2002, Park and Bera, 2009 or Fischer and Herrmann, 2010). Within this contribution we provide a comprehensive comparison between both different ME densities and their parametric competitors within different generalized GARCH models such as APARCH and GJR-GARCH.

Keywords and phrases: GARCH; APARCH; Entropy density; Skewness; Kurtosis

1 Introduction

Introduced by Engle (1982) and extended by Bollerslev (1986), generalized autoregressive conditional heteroskedasticity (GARCH) processes have become very popular as models for financial return data because they are able to capture both "distributional" stylized facts (e.g. thick tails and high peakedness) and stylized facts concerning the time structure (e.g. volatility clustering). Succeeding generalizations can be divided into three major classes: First of all, models that take asymmetric behaviour of volatility into account. Secondly, models with time-varying skewness and kurtosis rather than just time-varying variance and volatility, respectively. Thirdly, models based on a residual distribution different from the Gaussian one. It was already detected by Black (1976) that stock return volatility is strongly asymmetric: Negative returns are followed by larger increases in volatility than equally large positive returns. Both APARCH specification of Ding et al. (1993) which includes, among others, the TS-GARCH model (see Taylor, 1990 and Schwert, 1989 and 1990), the GJR-GARCH model (see Glosten et al., 1993) and the T-GARCH (see Zakoian, 1994) as special cases, and the (exponential) EGARCH model of Nelson (1991) account for this effect. In addition, there is no reason to assume that higher moments – in particular skewness and kurtosis represented by the third and fourth standardized moments – should be time-invariant. Allowing them to be time-varying may improve the approximation of the actual return distributions. Such types of models (so-called autoregressive conditional density (ARCD) models) were introduced by Hansen (1994). For other contributions see Harvey and Siddique (1999) or Hueng and McDonald (2005). Unfortunately, there is no systematic or significant evidence for time-varying higher moments. Finally, although GARCH models with conditionally normal errors imply leptokurtic unconditional distributions, Bollerslev (1987) found evidence that starting with leptokurtic and possibly skewed (conditional) error distributions will achieve better results. Bollerslev (1987), for instance, uses the Student-t distribution whereas Mittnik et al. (1998) advocate the stable distribution and Fischer (2004, 2006) found evidence in favour of generalized hyperbolic secant families. Parallel to these parametric distributions, recent years saw a rise in suggestions for maximum entropy (ME) distributions examined by econometricians, see e.g Rockinger and Jondeau (2002), Park and Bera (2009) or Fischer and Herrmann (2010), as well as by physicians, see. e.g. Quéiros (2005). All these approaches allow or can easily be extended to allow for kurtosis and skewness if presented in the unified framework following Herrmann (2009). Until now the ME approaches have only been compared with either the gaussian distribution (which for all suggestions appears as a special or limiting case) or with each other. The purpose of this contributions is to extend these new models to asymmetric volatility dynamics and to compare them to their most successful parametric peers. The presented approach is three-fold: Firstly, rather than compare plain GARCH models with MED density we use generalized variance specifications such as APARCH and GJR-GARCH. Secondly, a comprehensive comparison is provided between flexible parametric families and MED families. Thirdly, we compare the goodness-of-fit within the MED class.

2 A primer on GARCH models and its generalizations

Let $P_0, \dots P_T$ denote the time-discrete prices of an arbitrary asset from time $t = 0$ to $t = T$. Usually – rather than the prices themselves – the log-returns $R_1, \dots, R_T$ defined by $R_t \equiv \log(P_t/P_{t-1}) = \log(P_t) - \log(P_{t-1})$ are analyzed. In general, the standard models for the returns in financial econometrics are of the form

$$\Theta_m(L) R_t = \mu + U_t, \quad t = 1, \dots, T$$

with

$$U_t | F_{t-1} \sim D(0, h_t^2, \eta) \text{ or } U_t = h_t \epsilon_t \text{ with } \epsilon_t \sim D(0, 1, \eta), \tag{2.1}$$

where $\Theta_m(L)$ is a polynomial in the lag operator L of order m which allows to include linear dependence from the own history $R_{t-1}, \dots, R_{t-m}$ and $\mu \in \mathbb{R}$. Moreover, the distribution of the residuals U_t (conditioned on the information set $\mathcal{F}_{t-1}$ up to time $t-1$) is assumed to follow a standardized[1] distribution D with shape parameter η and time-varying variance h_t^2. For reasons of simplicity, assume that $\Theta_m(L) \equiv 1$ and $\mu \equiv 0$, i.e. $R_t = h_t \epsilon_t$. Otherwise,

[1]This means in particular that scale parameter and variance parameter h_t are identically.

replace R_t by $R_t^* \equiv \Theta_m(L)R_t - \mu$ in the sequel. In the GARCH$(1,1)$ specification of Bollerslev (1986),

$$h_t^2 = \alpha_0 + \alpha_1 R_{t-1}^2 + \beta_1 h_{t-1}^2 = \alpha_0 + \alpha_1 h_{t-1}^2 \epsilon_{t-1}^2 + \beta_1 h_{t-1}^2, \quad \alpha_0 > 0, \alpha_1, \beta_1 \geq 0, \tag{2.2}$$

where the fundamental ARCH model of Engle (1982) is included for $\beta_1 = 0$. GARCH models have been generalized in many different ways: In order to capture leverage effects (i.e. asymmetric behaviour of volatility for positive or negative returns), Zakoian (1994) introduced the T-GARCH model with standard deviation defined by

$$h_t = \alpha_0 + \alpha_1^+ R_{t-1}^+ - \alpha_1^- R_{t-1}^- + \beta_1 h_{t-1}, \quad \alpha_1^+ \geq 0, \alpha_1^- \geq 0, \tag{2.3}$$

where $R_t^+ \equiv \max\{R_t, 0\}$ and $R_t^- \equiv \min\{R_t, 0\}$. Imposing Box-Cox-transformations on both conditional standard deviation and asymmetric absolute returns essentially leads to the APARCH-specification of Ding et al. (1993), namely[2]

$$h_t^\lambda = \alpha_0 + \alpha_1(|R_{t-1}| - cR_{t-1})^\lambda + \beta_1 h_{t-1}^\lambda, \quad |c| \leq 1, \lambda \geq 0. \tag{2.4}$$

Equation (2.4) reduces to (2.3) for $\lambda = 1$, $\alpha_1 = \alpha_1^-/(2 - \alpha_1^+)$ and $c = 1 - \alpha_1^+(2 - \alpha_1^+)/\alpha_1^-$. Moreover, equation (2.2) is achieved for $\lambda = 2$ and $c = 0$. Restricting $\lambda = 2$ for the APARCH includes the GJR-GARCH model of Glosten, Jagannathan and Runkle (1993). Although there might be some further (theoretical) generalizations, using the APARCH specification for the variance equation will capture the conditional volatility sufficiently well.

3 Time-varying volatility based on maximum entropy

Contrary to parametric volatility models the maximum entropy models are characterized by the dynamics of the conditional moments of $U_t|F_{t-1}$ in time, e.g. as

$$E(U_t|F_{t-1}) = 0, \tag{3.1}$$

$$E(U_t^2|F_{t-1}) = h_t^2 = \alpha_0 + \alpha_1 R_{t-1}^2 + \beta_1 h_{t-1}^2, \tag{3.2}$$

but might also assume the more sophisticated volatility dynamics as presented above. Following Jaynes' (1957) principle of maximum entropy, the conditional distribution is chosen as the distribution which has maximal entropy within the set of distributions consistent to these conditional moments. Formally this may be written as

$$U_t|F_{t-1} \sim ME, \quad \text{with } f_{ME} = \underset{f}{argmax}\left(H(f), f \in \mathcal{D}\right), \tag{3.3}$$

where H denotes some entropy measure and $\mathcal{D}$ the set of distributions consistent with equations 3.1 and 3.2. Entropy H may be measured using some differentiable, convex function ϕ as

$$H(f) = -\int_{D(X)} \phi(f(x))dx, \tag{3.4}$$

[2]This model is sometimes also referred to as Power GARCH or P-GARCH model.

such that if $\mathcal{D}$ admits a density function f_{ME} with

$$\phi'(f_{ME}(x)) = \sum_{i=0}^{k} \lambda_i g_i(x), \tag{3.5}$$

it is at the same time the solution to the entropy maximization task above, see Kesavan and Kapur (1989). In these cases the solution's functional form is defined, but the λ_i have to be derived using numerical algorithms for dual problems. Such dual problems are given in Kapur (1994), for efficient algorithms see e.g. Rockinger and Jondeau (2002) or for generalized entropy measures Herrmann (2009).

The main advantage of the maximum entropy approach is its flexibility. The first source of flexibility is the choice of the measure for entropy. For the measures examined in this work see table 1.[3]

Author	$\phi(f)$	$f_{ME}(x)$	Parameter	Example
Shannon	$-f\ln(f)$	$\exp\left(-\sum_{i=0}^{k}\lambda_i g_i(x)\right)$	-	Gaussian
Havrda-Charvat	$\frac{1}{1-\alpha}$	$\left(\sum_{i=0}^{k}\lambda_i g_i(x)\right)^{\frac{1}{\alpha-1}}$	α	Student t
Kapur	$-f\ln(f)+$ $\frac{1}{c}(1+cf)\ln(1+cf)$	$\frac{1}{\exp\left(-\sum_{i=0}^{k}\lambda_i g_i(x)\right)-c}$	c	-

Table 1: Suggestions for Measures of Entropy.

Using the Havrda-Charvat entropy leads to stronger tails for $\alpha < 1$, the Kapur entropy for $c > 0$. Such that letting these freely adjust to the data gives flexibility with respect to kurtosis. Another way to include higher moments is the inclusion of additional restrictions for $\mathcal{D}$, e.g. as

$$E(tan^{-1}(U_t)|F_{t-1}) = m_3, \tag{3.6}$$

which implies a skewed set $\mathcal{D}$ or

$$E(\ln(1+U_t^2)|F_{t-1}) = m_4, \tag{3.7}$$

which implies a leptokurtic set $\mathcal{D}$.[4] Here m_3 and m_4 denote the target values for higher moments. Further suggestions for suitable moment functions have been compared in Park and Bera (2009), but we found that this combination performed best.

[3]The entropy measure suggested by Havdra-Charvat is a monotonic transformation of Rényi's entropy. In physics it is usually denoted as Tsallis entropy.

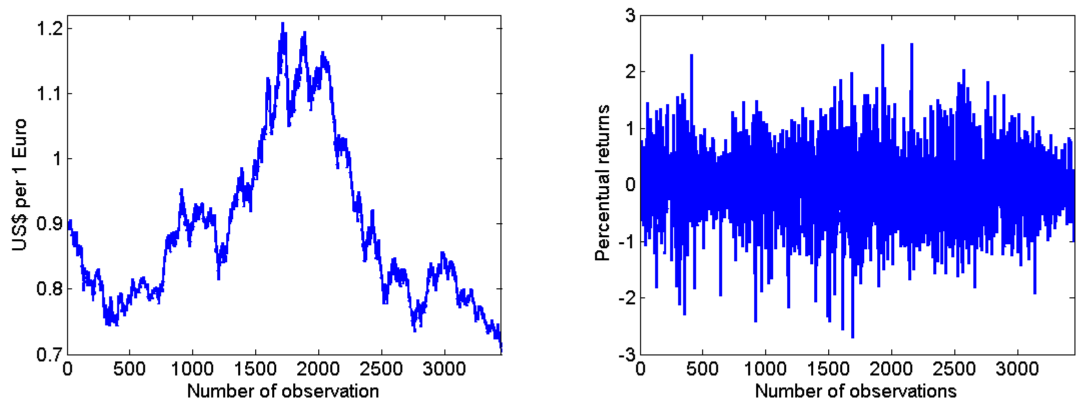
[4]For restrictions on possible moment combinations see e.g. Fischer and Herrmann (2010).

4 Application to fx data

4.1 Data set

We focus on the daily noon spot US dollar exchange rates (\$/local currency) for the Euro (EUR) over the period 3 January 1994 to 30 September 2007 (3453 observations)[5]. In a first step, the data are transformed to percentual log-returns defined as $R_{i,t} \equiv \ln(S_{i,t}/S_{i,t-1}) \cdot 100$. Both prices and log-returns can be seen in figure 1, below. To get some information on the

Figure 1: Prices and log-returns



(a) Exchange Rates (b) Log-returns

underlying data set, table 2 summarizes some basic descriptive statistics.

	$\hat{\mu}$	$\hat{s}$	$\mathbb{S}$	$\mathbb{K}$	$q_{0.25;0.75}$	$q_{0.4;0.6}$	JB	$Q(30)$	$Q^2(30)$
EUR	-0.0071	0.575	-0.1067	4.238	0.6619	0.2411	227.1*	26.51	291.4*

Table 2: Descriptive statistics of the data

In particular, the USD/EURO data set exhibits some skewness and leptokurtosis, measured by the third and fourth standardized moments $\mathbb{S}$ and $\mathbb{K}$. Moreover, the Jarque-Bera ($\mathcal{JB}$) test indicates non-normality. Finally, application of the Ljung-Box test to the returns (Q) and the squared returns (Q^2), respectively, suggests the presence of GARCH effects but no significant correlation between different returns.

[5]The data are available from `http://www.econ.queensu.ca/jae`.

4.2 Goodness-of-fit statistics

As a natural candidate for the goodness-of-fit we consider the maximum log-likelihood value ($\mathcal{LL}$) obtained from the ML estimation. It may be viewed as an overall measure of goodness-of-fit and allows to judge which candidate is more likely to have generated the data. To account for the different numbers of parameter k, we also calculate the Akaike information criterion (AIC), given by

$$\mathcal{AIC} = -2 \cdot \mathcal{LL} + \frac{2N(k+1)}{N-k-2},$$

where N denotes the number of data. An alternative penalization of additional parameter is given by the Bayesian information criterion as

$$\mathcal{BIC} = -2 \cdot \mathcal{LL} + k \cdot \ln N.$$

However, Boothe and Glassman (1987) presented arguments that non-nested model comparisons based on log-likelihood values may lead to spurious conclusions.

If the underlying model is correctly specified, the zero-mean and unit-variance returns $\widehat{\epsilon}_t|\mathcal{F}_{t-1} = (R_t - \widehat{\mu}_t)/\widehat{h}_t$ can be assumed to be independent realizations of the distribution $D(0,1,\widehat{\eta})$, where $\widehat{\eta}$ denotes the ML estimator of the shape vector η. Such that we can measure the distance between the fitted theoretical distribution and the empirical distribution of ϵ.

Wang et al. (2001) suggested the χ^2 goodness-of-fit statistic which can be calculated by

$$\chi^2 = \sum_{i=1}^{N_c} \frac{(H_i - F_i)^2}{F_i},$$

where H_i is the observed count frequency of ϵ in the $i-$th data class, F_i is the predicted count frequency under the assumed theoretical model and N_c is the number of classes.

Alternatively the Kolmogorov distance may be used as

$$\mathcal{KS} = 100 \cdot \sup_{x \in \mathbb{R}} |F_D(x) - \widehat{F}(x)|,$$

where F_D denotes the cumulative distribution function of $D(0,1,\widehat{\eta}_t)$ and $\widehat{F}$ the empirical distribution of ϵ.

Whereas $\mathcal{KS}$ emphasizes deviation around the median of the distribution, $\mathcal{AD}_0$ defined by

$$\mathcal{AD}_0 = \sup_{x \in \mathbb{R}} \frac{|F_D(x) - \widehat{F}(x)|}{\sqrt{F_D(x) \cdot (1 - F_D(x))}}$$

emphasizes discrepancies in the tails of the distribution. Instead of just the maximum deviation, one should also have a look at the second and the third largest value, denoted by $\mathcal{AD}_1$ and $\mathcal{AD}_2$.

4.3 Parametric competitors

For a general comprehensive overview on successful parametric distribution family we refer to Bao, Lee and Saltoglu (2004) or Fischer (2010). Within our empirical analysis we focus on popular multi-parametric distributions which have already been successfully applied to financial return data. Among them, the Student-t distribution (T) and the generalized Student-t (GT) distribution of McDonald and Newey (1988) together with its skew counterpart (SGT2) developed by Grottke (2001). In contrast, McDonald (1991), and McDonald and Bookstaber (1991) used the exponentially generalized beta of the second kind (EGB2) distribution which generalizes the logistic distribution in a natural way. Similarly, the less-known but rather flexible generalized secant hyperbolic (GSH) distribution of Vaughan (2002) and its skew generalization of Fischer (2004) is taken into consideration. In addition, Theodossiou's (2000) skewed family of the generalized error distribution (SGED). Last, but not least we included the inverse hyperbolic sine (IHS) distribution which is used successfully, for instance, in Choi (2001) to model asymmetric and fat-tailed distributions.

4.4 Empirical Results

The results of the maximum likelihood estimations are summarized in table 3 and 4. For reasons of brevity, only the different goodness-of-fit measures were reported herein. Results for the specific parameter estimators for the different distribution families are available from the authors by request. First of all, the highest log-likelihood LL and the lowest of the other goodness-of-fit measures (representing the most favourable choice) were marked bold for each of the four generalized GARCH models (plain GARCH, T-GARCH, APARCH and GJR-GARCH). Across all variance specification, the results concerning the order of the distribution families remains nearly constant.

Within the classical parametric families, both SGT2 and SGSH are pre-dominant if only likelihood is taken into account. The logistic distribution gives by far the best results if additional parameters are penalized. For the likelihood-based measures all three MED densities are outperformed. However, if we focus on the tail-related Anderson-Darling statistics MEHC dominates all other competitors while MEK exhibits the minimal χ^2-value and Kolmogorov-Smirnov distance, respectively.

Within the entropy density, MED minimizes the log-likelihood and maximizes AIC and BIC. Above that conditional non-normality in the sense of Bollerslev (1987) is rejected for nearly all multi-parametric families. Finally, the close theoretical relation between the Student-t distribution and the MEHC distribution is recovered for the underlying data set.

5 Summary

This work extends the new approaches of maximum entropy volatility models to more sophisticated volatility dynamics and compares these to the the to most flexible parametric models known in econometrics. Using maximum entropy specifications with similar flexibility with respect to skewness and kurtosis we find that the maximum entropy densities give results similar to their parametric peers as far as overall fit is concerned. But if we compare the fit in the tails, only the Havrda-Charvat density – that may be related to t-type distributions – competes with its parametric peers.

References

[1] Bao, Y., Lee, T. and B. Saltoğlu (2004): *A test for density forecast comparison with application to risk management.* Working Paper, Department of Economics, University of California.

[2] Black, F.(1976): *Studies of Stock Price Volatility Changes.* Proceedings of the 1976 Business and Economics Statistics Section, American Statistical Association: 177-181.

[3] Bollerslev, T. (1986): Generalized Autoregressive Conditional Heteroskedasticity. *Journal of Econometrics* 31:307-327.

[4] Bollerslev, T. (1987): A Conditionally Heteroskedastic Time Series Model for Speculative Prices and Rates of Return. *Review of Economics and Statistics*, 69(3):542-547.

[5] Boothe, P., and Glassman, D. (1987), The Statistical Distribution Of Exchange Rates, *Journal of International Economics*, May, pp. 297-319.

[6] Choi, P. (2001): *Estimation of value-at-risk using Johnson S_U-normal distribution.* Working Paper, Department of Economics, Texas A&M University.

[7] Cover, T. M., and J.A. Thomas (2006): *Elements of Information Theory*, New York: John Wiley & Sons.

[8] Ding, Z. and C. W. J. Granger and R. F. Engle (1993): A long memory property of stock markets returns and a new model. *Journal of Empirical Finance* 1(1):83-106.

[9] Engle, R. F. (1982): Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of the United Kingdom Inflation. *Econometrica* 50(4):987-1007.

[10] Fischer, M. (2004): Skew generalized secant hyperbolic distributions: unconditional and conditional fit to asset returns, *Austrian Journal of Statistics* **33**(3), 293-304.

[11] Fischer, M. (2006): The skew generalized secant hyperbolic family, *Austrian Journal of Statistics* **35**(4), 437-444.

[12] Fischer, M. (2010): Financial return distributions. In: M. Lovric, International Encyclopedia of Statistical Sciences, Springer.

[13] Fischer, M. and K. Herrmann (2010): "An alternative maximum entropy model for time-varying moments with application to financial returns", *Studies in Nonlinear Dynamics & Econometrics*, to appear in **14**(3) or **14**(4).

[14] Glosten, L. and R. Jagannathan and D. E. Runkle (1993): On the Relation between the Expected Value and the Volatility of the Nominal Excess Return on Stocks. *Journal of Finance* 48(5):1779-1801.

[15] Grottke, M. (2001): Die t-Verteilung und ihre Verallgemeinerungen als Modell für Finanzmarktdaten. Josef Eul, Köln.

[16] Hansen, B. (1994): Autoregressive Conditional Density Estimation, *International Economic Review*, 35, 705730.

[17] Harvey, C. R. and A. Siddique (1999): *Autoregressive Conditional Skewness. Journal of Financial and Quantitative Analysis* 34(4):465-487, 1999.

[18] Herrmann, K. (2009): Non-Extensitivity vs. Informative Moments for Financial Models - A Unifying Framework and Empirical Results, *EPL*, **88**, 30007.

[19] Hueng, C. J. and J. B. Mc Donald: (2005) *Forecasting Asymmetries in Stock Returns: Evidence from Higher Moments and Conditional Densities*. Working Paper, Department of Economics Western, Michigan University, 2003.

[20] Jaynes, E. T. (1957): "Information Theory and Statistical Mechanics", *Physical Review*, 106(4), 620-630.

[21] Kapur J.N. (1994): Measures of Information and their Applications, *Wiley Eastern Limited*, New Delhi.

[22] Kesavan, H. K. and J. N. Kapur (1989): "The Generalized Maximum Entropy Principle", *IEEE Transactions on Systems, Man and Cybernetics*, **19**(5), 1042-1052.

[23] McDonald, J. B. (1991): Parametric models for partially adaptive estimation with skewed and leptokurtic residuals. *Economics Letters* **37**(3): 273-278.

[24] McDonald, J. B.; Bookstaber, R. M. (1991): Option pricing for generalized distributions. *Communications in Statistics - Theory and Methods* **20**(12): 4053-4068.

[25] McDonald, J. B.; Newey, W. K. (1988): Partially adaptive estimation of regression models via the generalized t distribution. *Econometric Theory* **4**(3): 428-457.

[26] Mittnik, S. and M. S. Paolella and S. T. Rachev (1998): Unconditional and Conditional Distribution Models for the Nikkei Index. *Asia-Pacific Financial Markets* 5(2):99-128.

[27] Nelson, D. B. (1991): Conditional Heteroskedasticity in Asset Returns: A New Approach. *Econometrica* 59(2):347-370.

[28] Park, S. Y., and A. K. Bera (2009): "Maximum Entropy Autoregressive Conditional Heteroskedasticity Model", *Journal of Econometrics*, **150**(2), 219-230.

[29] Quéiros, S.M.D. (2005): "On non-Gaussianity and dependence in financial time series: a nonextensive approach", *Quantitative Finance*, **5**(5), 475 487.

[30] Rockinger, M. and E. Jondeau (2002): "Entropy Densities with an Application to Autoregressive Conditional Skewness and Kurtosis", *Journal of Econometrics*, 106, 119-142.

[31] Schwert, G. W. (1989): Why does Stock Market Volatility change over Time?. *Journal of Finance* 44(5):1115-1153.

[32] Schwert, G. W. (1990): Stock Volatility and the Crash of '87. *Review of Financial Studies* 3(1):77-102.

[33] Taylor, S. (1990): *Modelling Financial Time Series*. Wiley, New York.

[34] Theodossiou, P.: *Skewed Generalized Error Distribution of Financial Assets and Option Pricing*. Working Paper, School of Business, Rutgers University, 2000.

[35] Vaughan, D. C. (2002): The generalized secant hyperbolic family and its properties, *Communications in Statistics - Theory and Methods* **31**(2), 219-238.

[36] Wang, K. and C. Fawson and C. B. Barett and J. B. McDonald (2001): A Flexible Parametric GARCH Model with an Application to Exchange Rates. *Journal of Applied Econometrics* 16:521-536, 2001.

[37] Zakoian, J. M. (1994): Threshold Heteroskedastic Models. *Journal of Economic Dynamics & Control* 18(5):931-955.

GARCH(1,1)

Distribution	k	$\mathcal{LL}$	$\mathcal{AIC}$	$\mathcal{BIC}$	KS	χ^2	AD_0	AD_1	AD_2
NORM	4	-2877.5	5765	5787.6	2.94	48.023	3.061	0.29	0.264
t	5	-2830.9	5673.8	5702.5	1.43	8.693	0.04	0.039	0.038
GT	6	-2829.1	5672.2	5707.1	1.27	6.115	0.052	0.05	0.05
SGT2	7	**-2828.2**	5672.4	5713.4	1.09	6.848	0.051	0.045	0.044
LOG	4	-2829.9	**5669.8**	**5692.4**	1.42	6.968	0.057	0.046	0.045
EGB2	6	-2829.6	5673.2	5708.1	1.38	7.23	0.051	0.041	0.04
SGED	6	-2829.8	5673.6	5708.5	1.1	10.187	0.122	0.06	0.058
IHS	6	-2830.3	5674.6	5709.5	1.4	7.552	0.04	0.038	0.038
GSH	5	-2829.8	5671.6	5700.3	1.4	6.631	0.054	0.044	0.043
SGSH	6	-2828.3	5670.6	5705.5	1.01	6.272	0.043	**0.033**	**0.032**
MED	6	-2829.4	5672.8	5707.7	1.01	5.383	0.139	0.05	0.048
MEHC	6	-2829.9	5673.8	5708.7	1.09	8.038	**0.035**	0.034	0.034
MEK	6	-2830.7	5675.4	5710.3	**0.93**	**4.776**	0.237	0.064	0.06

Threshold GARCH

Distribution	k	$\mathcal{LL}$	$\mathcal{AIC}$	$\mathcal{BIC}$	KS	χ^2	AD_0	AD_1	AD_2
NORM	5	-2878.7	5769.4	5798.1	2.92	47.426	2.402	0.368	0.298
t	6	-2832.5	5679	5713.9	1.54	11.252	0.05	0.045	**0.039**
GT	7	-2830.6	5677.2	5718.2	1.43	**7.912**	0.057	0.052	0.046
SGT2	8	-2830.1	5678.3	5725.4	1.24	8.563	0.049	0.047	0.045
LOG	5	-2831.5	**5675**	**5703.7**	1.53	8.84	0.052	0.049	0.047
EGB2	7	-2831.4	5678.8	5719.8	1.52	9.629	0.051	0.048	0.045
SGED	7	-2831.6	5679.2	5720.2	1.18	9.766	0.111	0.057	0.057
IHS	7	-2831.8	5679.6	5720.6	1.59	10.532	0.051	0.046	0.042
GSH	6	-2831.4	5676.8	5711.7	1.51	8.511	0.049	0.047	0.045
SGSH	7	**-2829.7**	5675.4	5716.4	1.16	10.777	0.041	0.041	0.041
MED	7	-2830.5	5677	5718	1.22	9.176	0.116	0.05	0.047
MEHC	7	-2831.5	5679	5720	1.25	12.233	**0.04**	**0.04**	**0.039**
MEK	7	-2831.9	5679.8	5720.8	**1.14**	8.585	0.197	0.071	0.063

Table 3: Comparison of goodness-of-fit measures (I)

APARCH

Distribution	k	$\mathcal{LL}$	$\mathcal{AIC}$	$\mathcal{BIC}$	KS	χ^2	AD_0	AD_1	AD_2
NORM	6	-2876.6	5767.2	5802.1	2.89	48.863	2.958	0.293	0.284
t	7	-2830.2	5676.4	5717.4	1.52	9.971	0.042	0.041	0.04
GT	8	-2828.4	5674.9	5722	1.34	7.227	0.05	0.05	0.05
SGT2	9	**-2826.7**	5673.5	5726.7	1.19	8.758	0.05	0.046	0.045
LOG	6	-2829.2	**5672.4**	**5707.3**	1.5	7.878	0.055	0.046	0.046
EGB2	8	-2829.1	5676.3	5723.4	1.46	**7.053**	0.051	0.042	0.042
SGED	8	-2829.4	5676.9	5724	1.17	10.846	0.122	0.058	0.058
IHS	8	-2829.5	5677.1	5724.2	1.49	8.498	0.042	0.041	0.041
GSH	7	-2829.1	5674.2	5715.2	1.49	7.613	0.053	0.044	0.044
SGSH	8	-2827.5	5673.1	5720.2	1.09	7.652	0.044	**0.037**	**0.037**
MED	8	-2828.6	5675.3	5722.4	1.15	7.36	0.134	0.047	0.046
MEHC	8	-2829.2	5676.5	5723.6	1.15	9.123	**0.038**	**0.037**	**0.037**
MEK	8	-2830	5678.1	5725.2	**1.06**	7.231	0.231	0.061	0.058

GJR_GARCH

Distribution	k	$\mathcal{LL}$	$\mathcal{AIC}$	$\mathcal{BIC}$	KS	χ^2	AD_0	AD_1	AD_2
NORM	5	-2877.2	5766.4	5795.1	2.97	47.667	3.074	0.283	0.257
t	6	-2830.5	5675	5709.9	1.49	9.214	0.044	0.038	0.037
GT	7	-2828.7	5673.4	5714.4	1.34	6.938	0.057	0.051	0.05
SGT2	8	-2828.1	5674.3	5721.4	1.15	7.438	0.05	0.05	0.043
LOG	5	-2829.5	**5671**	**5699.7**	1.47	7.488	0.056	0.052	0.046
EGB2	7	-2829.4	5674.8	5715.8	1.44	7.642	0.051	0.046	0.04
SGED	7	-2829.7	5675.4	5716.4	1.15	11.262	0.122	0.064	0.057
IHS	7	-2829.9	5675.8	5716.8	1.48	8.42	0.046	0.04	0.038
GSH	6	-2829.5	5673	5707.9	1.45	7.146	0.054	0.05	0.043
SGSH	7	**-2827.9**	5671.8	5712.8	1.06	6.755	0.043	0.037	**0.034**
MED	7	-2829.1	5674.2	5715.2	1.11	7.273	0.138	0.056	0.048
MEHC	7	-2829.6	5675.2	5716.2	1.13	8.295	**0.039**	**0.035**	**0.034**
MEK	7	-2830.4	5676.8	5717.8	**1.02**	**6.115**	0.237	0.068	0.061

Table 4: Comparison of goodness-of-fit measures (II)

Diskussionspapiere 2010
Discussion Papers 2010

01/2010 **Mosthaf, Alexander, Schnabel, Claus and Stephani, Jens:** Low-wage careers: Are there dead-end firms and dead-end jobs?

02/2010 **Schlüter, Stephan and Matt Davison:** Pricing an European Gas Storage Facility using a Continuous-Time Spot Price Model with GARCH Diffusion

Diskussionspapiere 2009
Discussion Papers 2009

01/2009 **Addison, John T. and Claus Schnabel:** Worker Directors: A German Product that Didn't Export?

02/2009 **Uhde, André and Ulrich Heimeshoff:** Consolidation in banking and financial stability in Europe: Empirical evidence

03/2009 **Gu, Yiquan and Tobias Wenzel:** Product Variety, Price Elasticity of Demand and Fixed Cost in Spatial Models

04/2009 **Schlüter, Stephan:** A Two-Factor Model for Electricity Prices with Dynamic Volatility

05/2009 **Schlüter, Stephan and Fischer, Matthias**: A Tail Quantile Approximation Formula for the Student t and the Symmetric Generalized Hyperbolic Distribution

06/2009 **Ardelean, Vlad:** The impacts of outliers on different estimators for GARCH processes: an empirical study

07/2009 **Herrmann, Klaus:** Non-Extensitivity versus Informative Moments for Financial Models - A Unifying Framework and Empirical Results

08/2009 **Herr, Annika:** Product differentiation and welfare in a mixed duopoly with regulated prices: The case of a public and a private hospital

09/2009 **Dewenter**, Ralf, **Haucap**, Justus and **Wenzel**, Tobias: Indirect Network Effects with Two Salop Circles: The Example of the Music Industry

10/2009 **Stuehmeier**, Torben and **Wenzel**, Tobias: Getting Beer During Commercials: Adverse Effects of Ad-Avoidance

11/2009 **Klein**, Ingo, **Köck**, Christian and **Tinkl**, Fabian: Spatial-serial dependency in multivariate GARCH models and dynamic copulas: A simulation study

12/2009 **Schlüter**, Stephan: Constructing a Quasilinear Moving Average Using the Scaling Function

13/2009 **Blien,** Uwe, **Dauth,** Wolfgang**, Schank,** Thorsten and **Schnabel,** Claus**:** The institutional context of an "empirical law": The wage curve under different regimes of collective bargaining

14/2009 **Mosthaf**, Alexander, **Schank**, Thorsten and **Schnabel**, Claus: Low-wage employment versus unemployment: Which one provides better prospects for women?

Diskussionspapiere 2008
Discussion Papers 2008

01/2008 **Grimm, Veronika and Gregor Zoettl**: Strategic Capacity Choice under Uncertainty: The Impact of Market Structure on Investment and Welfare

02/2008 **Grimm, Veronika and Gregor Zoettl**: Production under Uncertainty: A Characterization of Welfare Enhancing and Optimal Price Caps

03/2008 **Engelmann, Dirk and Veronika Grimm**: Mechanisms for Efficient Voting with Private Information about Preferences

04/2008 **Schnabel, Claus and Joachim Wagner**: The Aging of the Unions in West Germany, 1980-2006

05/2008 **Wenzel, Tobias**: On the Incentives to Form Strategic Coalitions in ATM Markets

06/2008 **Herrmann, Klaus**: Models for Time-varying Moments Using Maximum Entropy Applied to a Generalized Measure of Volatility

07/2008 **Klein, Ingo and Michael Grottke:** On J.M. Keynes' "The Principal Averages and the Laws of Error which Lead to Them" - Refinement and Generalisation